

**Essays on Consumer Bankruptcy, Employment, Firm  
Dynamics, and Aging in the United States**

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**Gajendran Raveendranathan**

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**Timothy Kehoe, Advisor  
Manuel Amador, Co-Advisor**

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# Dedication

To those who held me up over the years

## Abstract

This dissertation consists of four chapters. The unifying topic of these chapters is the study of recent macroeconomic trends in consumer bankruptcy, employment, firm dynamics, and aging in the U.S. economy.

The first chapter shows that improved matching between borrowers and lenders quantitatively explains the rise in unsecured credit and consumer bankruptcies in the United States. The model is calibrated to match the increase in the population with access to unsecured credit. Results show that the increase in the matching efficiency accounts for more than 60 percent of the rise in unsecured credit and 80 percent of the rise in consumer bankruptcies. Furthermore, this explanation is consistent with the observed behavior of measures such as the charge-off rate, the (cross-sectional) average spread, and the increase in credit access by income quintiles.

The second chapter quantifies the effect of investment-specific productivity and labor augmenting productivity in causing the decrease in low-skilled manufacturing employment concentrated in recessions. When the transition is computed with only investment-specific productivity, 18 percent of the decrease in low-skilled manufacturing employment is observed during recessions. When the transition is computed with both investment-specific productivity and labor-augmenting productivity, 52 percent of the decrease is observed during recessions.

In the third chapter, written jointly with Joao Ayres, we show that the drop in firm entry has been an important feature of the Great Recession. In a standard model of firm dynamics featuring aggregate uncertainty and firm heterogeneity, we show that a negative aggregate productivity shock does not generate a drop in firm entry, while a negative demand shock does. We also provide empirical evidence that contradicts common explanations for the lack of firm entry, such as financial constraints, offshoring, increased uncertainty at the firm level, and increased self-employment.

The fourth chapter shows that the aging population in the U.S. economy can jointly account for the following 3 facts since 2007: 1) Lack of recovery in employment to population for ages 25-64 or 25 plus; 2) Recovery in employment to labor force for ages 25-64 and 25 plus; and 3) Faster recovery of labor productivity compared to GDP.

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# Chapter 1

## The Rise in Unsecured Credit and Consumer Bankruptcies

### 1.1 Introduction

The United States has experienced an unprecedented increase in unsecured credit in the last four decades. Unsecured credit refers to consumer credit that is not secured by collateral; it mainly consists of credit card debt. Between 1970 and 2004, unsecured credit increased sixteen-fold from 0.4 to 6.4 percent of GDP. The magnitude of the rise is further emphasized by the fact that during the same period, secured consumer credit — excluding real estate — remained stable at 11 percent of GDP. The rise in unsecured credit was associated with a contemporaneous rise in consumer bankruptcies. Consumer bankruptcies increased six-fold from 0.1 to 0.6 percent of working-age population.

This paper makes a two-fold contribution in explaining the rise in unsecured credit and consumer bankruptcies. The first contribution is to show that improved matching between borrowers and lenders quantitatively accounts for the above phenomenon. The hypothesis is that due to the IT revolution, it is relatively easier for a borrower to find a lender and vice versa. In a model of unsecured credit, this change is studied as an increase in the matching efficiency of directed search. It alleviates a search friction between borrowers and lenders, which increases the population with unsecured credit access — a fact observed for the United States. Therefore, the model is calibrated to match the rise in unsecured credit access; and is shown to be consistent with the rise in

unsecured credit, consumer bankruptcies, and the observed behavior of measures such as the charge-off rate and the (cross-sectional) average spread.<sup>1</sup>

The second contribution is to explore three alternative explanations, and show that these explanations also increase unsecured credit and consumer bankruptcies. However, these explanations are not consistent with the observed behavior of the (cross-sectional) average spread. The three alternative explanations are as follows: (1) a decrease in the cost of bankruptcy; (2) a decrease in the lending fee; and (3) an increase in the lender's information about the borrower's characteristics.

Given the unsecured nature of credit, a difficulty faced by standard theory is quantitatively accounting for the rise in consumer bankruptcies. For example, Livshits, MacGee, and Tertilt (2010) study various explanations in a model of unsecured credit and find that only a decrease in the cost of bankruptcy generates a rise that is consistent with the data. This paper incorporates the following feature of the U.S. legal system to their framework: the court can dismiss a bankruptcy filing (discharge of debt) if the debt is primarily consumer debt and if it can be repaid in three to five years. As this paper shows, the incorporation of this feature leads to a quantitatively consistent increase in consumer bankruptcies for not only a decrease in the cost of bankruptcy, but also for other possible explanations, including improved matching. Subsequently this paper analyzes implications for measures beyond credit and bankruptcies, and shows that improved matching is the explanation that is also consistent with the observed behavior of the (cross-sectional) average spread.

The intuition for an increase in the matching efficiency (improved matching) is as follows. In the model, there is a search friction that prevents agents from receiving immediate access to credit. Lenders incur a fixed cost to send credit offers to submarkets. A submarket is comprised of agents with a given level of assets and productivity. By assumption, a lender makes profits only when the agent borrows. An increase in the matching efficiency increases the probability of credit access across all submarkets, especially for those that are most profitable. The most profitable submarkets are the ones where agents are most likely to sustain higher levels of debt. Therefore, increased matching efficiency disproportionately increases credit access of agents who are more

---

<sup>1</sup> The charge-off rate is the total percent of debt written off. The spread is the difference between the credit card interest rate and the risk-free rate.

likely to borrow. This leads to an increase in credit and bankruptcies that is quantitatively consistent with the data.

As mentioned above, the increase in matching efficiency is also consistent with additional evidence on the charge-off rate and the (cross-sectional) average spread. The charge-off rate increases because the economy transitions from an initial steady state without unsecured credit to one with unsecured credit.<sup>2</sup> That is, agents start in the initial steady state without debt. Then, when they are hit with medium shocks to productivity, they start to borrow. If these shocks persist, the agents accumulate debt. With more debt, they are more likely to default. Thus a higher percent of debt is written-off, which increases the charge-off rate.

An increase in the matching efficiency leads to a small change in the (cross-sectional) average spread for the following reason: as mentioned above, it disproportionately increases access to agents who are the most profitable. This has little impact on the average because there are more borrowers from the same pool. However, an increase in the matching efficiency also increases credit access of agents who are less profitable. Results show that these are agents who have relatively higher persistent productivities. As a result, these agents are relatively safer compared to the average. An increase in the mass of these agents puts downward pressure on the average spread. However, as these agents accumulate debt, they also become more risky which puts upward pressure on the average spread. As a result increased matching efficiency leads to a small change in the (cross-sectional) average spread, consistent with the data.

As mentioned above, this paper also explores three alternative explanations for the rise in unsecured credit and consumer bankruptcies. The first alternative explanation is a decrease in the cost of bankruptcy. This is studied as a decrease in the utility cost of bankruptcy, also referred to as *stigma* by Athreya (2004) and Livshits, MacGee, and Tertilt (2010). It is a measure of the non-pecuniary cost associated with filing for bankruptcy. For example, Gross and Souleles (2002) estimate that stigma has decreased. The second alternative explanation is a decrease in an exogenous lending fee. The exogenous lending fee is an approximation for a markup over the risk-free rate in the market for unsecured credit. Livshits, MacGee, and Tertilt (2010) estimate a decrease

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<sup>2</sup> The model is calibrated along the transition path to match unsecured credit access in 1970 and 2004. However, the transition is assumed to have started in 1951 with no unsecured credit, which is also the year of the first universal credit card.

for the fee between early 1980s and late 1990s. Increased competition in the market for unsecured credit can be cited as a possible reason for a decrease in the lending fee.

Without the legal feature, Livshits, MacGee, and Tertilt (2010) find that a decrease in the cost of bankruptcy increases bankruptcies, but not credit. A decrease in the lending fee increases credit, but not bankruptcies. As this paper shows, with the legal feature, a decrease in cost of bankruptcy or a decrease in the lending fee increases both credit and bankruptcies. However, these explanations are not consistent with the observed behavior of the (cross-sectional) average spread.

A decrease in the cost of bankruptcy increases the incentive for agents to take on levels of debt with a higher default probability. It leads to more borrowing and bankruptcies. However, it overestimates the increase in the average spread for the same reason: agents are more willing take on levels of debt with a higher default probability. That is, a higher default probability increases the lending rate which increases the spread between the lending rate and the risk-free rate.

A decrease in the lending fee decreases the cost of borrowing. Therefore, it also leads to an increase in borrowing and bankruptcies. However, it leads to a decrease in the average spread because it decreases the lending rate and, subsequently the spread between the lending rate and the risk-free rate. The combination of a decrease in the stigma and a decrease in the lending fee does lead to a small change in the average spread, consistent with the data. The only alternative explanation that does equally well compared to an increase in the matching efficiency is the combination of a decrease in the stigma and the lending fee.

Finally, this paper extends the model to explore a third alternative explanation: an increase in the lender's information about the borrower's characteristics. An increase in the use of credit scoring can be cited as a possible reason for this explanation. While this explanation also leads to a rise in credit and bankruptcies, it leads to a decrease in the (cross-sectional) average spread. This is because agents with medium shocks to earnings, who were hurt otherwise by the lenders belief that they might be less productive, borrow at lower spreads.



## Related literature

This paper builds on the framework proposed by Chatterjee, Corbae, Nakajima, Rios-Rull (2006), Livshits, MacGee, and Tertilt (2007), and Herkenhoff (2015). The first two papers incorporate unsecured credit and default to an incomplete markets model with idiosyncratic earnings risk and non-contingent bonds. The latter incorporates a search friction between borrowers and lenders. A novel feature in this paper is the incorporation of the legal feature discussed above. This feature turns out to be important because it results in a quantitatively consistent increase in consumer bankruptcies for not only a decrease in the cost of bankruptcy, but also for other explanations discussed above.

Herkenhoff (2015) and Drozd and Nosal (2008) also study an alleviation of a search friction between borrowers and lenders. This paper builds on these two papers in two dimensions. First, it incorporates the legal feature mentioned above, and shows that an increase in the matching efficiency quantitatively explains the rise in consumer bankruptcies. Second, this paper shows that an increase in the matching efficiency is consistent with the observed behavior for the charge-off rate and the (cross-sectional) average spread.

This paper is also related to the literature that proposes explanations that depart from a search friction. As discussed above, Athreya (2004) and Livshits, MacGee, and Tertilt (2010) study a decrease in the cost of bankruptcy and a decrease in the lending fee. Narajabad (2012), Athreya, Tam, and Young (2012), and Sanchez (2012) propose information friction based explanations. Livshits, MacGee, and Tertilt (2015) study both information frictions and the fixed cost of lending contracts.

This paper is organized as follows. Section 2 discusses the empirical motivation. Section 3 describes the benchmark model with endogenous credit access. Section 4 discusses functional forms and parameterization. Section 5 discusses the results for an increase in matching efficiency. Section 6 extends the model to study alternative explanations. Section 7 concludes.

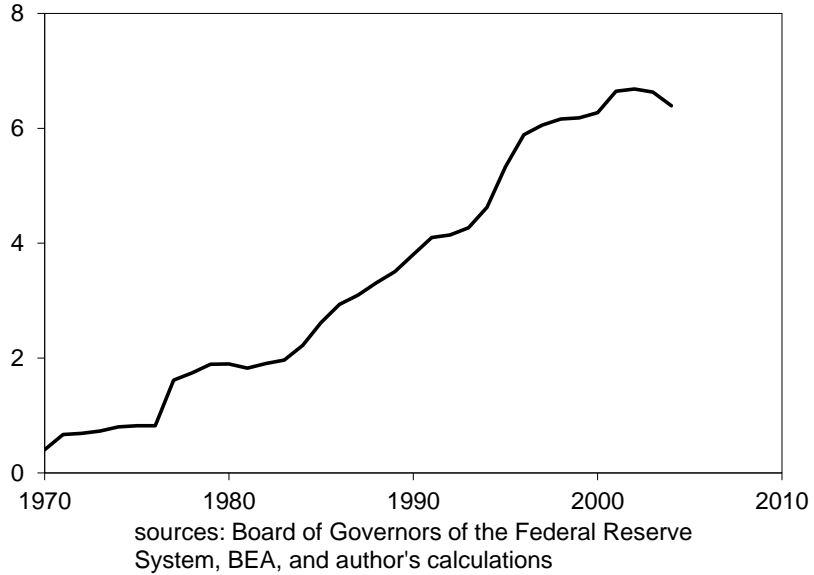
## 1.2 Data

In the United States, personal bankruptcy comprises two types of bankruptcies: Chapter 7 and Chapter 13. Chapter 7 is a liquidation bankruptcy in which filing individuals

lose non-exempt assets while debt is discharged. Under Chapter 13, which is a re-organization bankruptcy, individuals keep their assets, but have to repay debt from future income. Following the literature, this paper focuses on Chapter 7 bankruptcies because individuals with unsecured debt and zero non-exempt assets file chapter 7.

Figure 1.1 shows the sixteen-fold increase in unsecured credit. Figure 1.2 shows the six-fold increase in consumer bankruptcies.<sup>3</sup>

Figure 1.1: **Rise in unsecured credit (percent of GDP)**

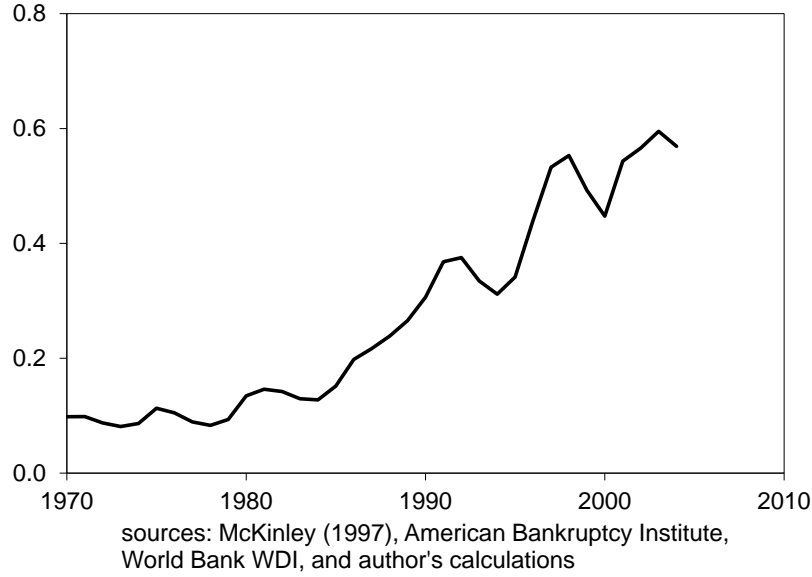


### 1.3 Benchmark model

The basic framework is as follows: infinitely-lived agents face idiosyncratic earnings risk and supply inelastic labor. The idiosyncratic state  $(b, \lambda, i)$  is given by the level of debt

<sup>3</sup> This paper does not analyze the U.S. economy after 2004 because of the bankruptcy policy reform in 2005 and the Great Recession from 2007-2009.

Figure 1.2: **Rise in consumer bankruptcies (percent of working-age population)**



$b$ , the vector of productivity components  $\lambda$ , and the credit standing  $i$ .

A negative  $b$  indicates the asset level and a positive  $b$  indicates the debt level. The vector of productivity components is comprised of a persistent, transitory, and permanent component where  $\lambda = (\gamma, z, \eta_0)$ . The productivity for the current-period is given by  $\nu_\lambda = \exp(\gamma + z + \eta_0)$ . Credit standing  $i$  takes on two values,  $C$  and  $N$ .  $C$  denotes an agent with credit access, and  $N$  denotes an agent without credit access.

As discussed above, in the United States, discharge of consumer debt is dismissed if the individual is able to repay the debt in three to five years Ruttenberg (2012).<sup>4</sup>

The model approximates the above criterion to a one-year test and assumes that an agent cannot file for bankruptcy if expected earnings net of allowed expenses are greater

<sup>4</sup> This is a feature of the legal system that prevents abuse of Chapter 7 relief. For example, there is evidence for this feature due to the Bankruptcy Amendments and Federal Judgeship Act of 1984. This is a criterion that applies to consumer debt; not medical debt. Therefore, the model abstracts from medical expense shocks.

than 20 percent of debt. That is, given  $(b, \lambda)$ , an agent cannot file for bankruptcy if

$$\underbrace{E_{\lambda'|\lambda}(\nu'_{\lambda'})w_{t+1}}_{\text{expected earnings}} - \underbrace{(ew_{t+1})}_{\text{allowed expenses}} > 0.2b,$$

where  $E_{\lambda'|\lambda}(\nu'_{\lambda'})$  is the expected effective productivity for the next period given  $\lambda$ ;  $w_{t+1}$  is the average wage in the economy for the next period; and  $e \in [0, 1]$  is an exogenous parameter that captures the fraction of the average wage allowed for expenses.

Figure 1.3: **Illustration of agents that can file for bankruptcy**

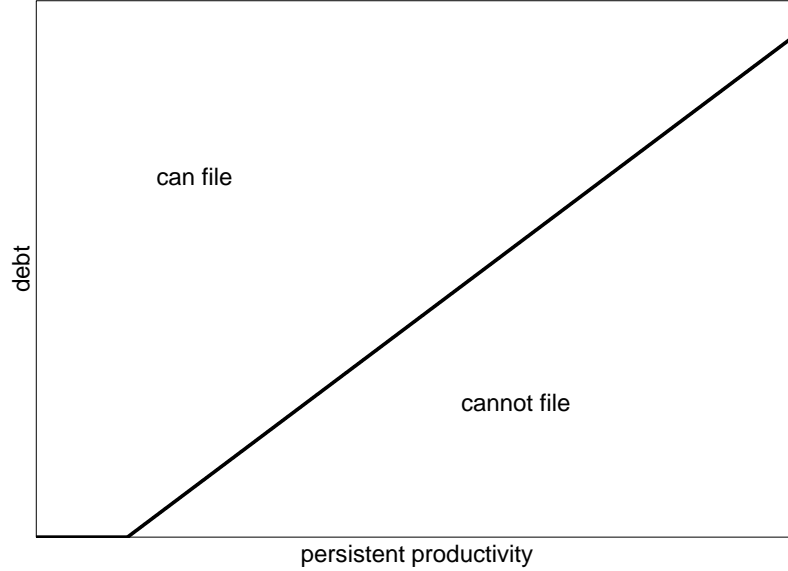


Figure 1.3 presents a graphical illustration of this approximation. An agent with expected earnings less than the allowed expenses can file for any level of debt. These are agents with the lowest persistent productivities — the flat part of the line in Figure 1.3. Agents with higher persistent productivities have higher expected earnings. Therefore, these agents need a higher level of debt to be eligible to file for bankruptcy.

An agent who does not default can choose to borrow or save and retains credit access for the next period. An agent who defaults loses credit access by breaking the match

with the lender and is not allowed to save during the default period. That is, the agent just consumes the labor earnings.<sup>5</sup>

A search friction prevents agents from receiving immediate credit access after breaking a match with the lender. Without credit access, the agent can save, but not borrow. That is, the agent is restricted to  $b' \leq 0$ . The structure of the search friction is the same as in Herkenhoff (2015). Lenders direct search to submarkets  $(b', \lambda') \in [B \times \Lambda]$ . Let  $v_t(b, \lambda)$  be the mass of credit offers and  $u_t(b, \lambda)$  be the mass of borrowers in submarket  $(b, \lambda)$ . The borrower's probability of credit access in submarket  $(b, \lambda)$  is given by the following expression:

$$p_t(b, \lambda) = \frac{A_t M(u_t(b, \lambda), v_t(b, \lambda))}{u_t(b, \lambda)},$$

where  $A_t$  is a parameter capturing the matching efficiency and  $M(u, v)$  is the matching function.  $A_t M(u_t(b, \lambda), v_t(b, \lambda))$  is the mass of matches in submarket  $(b, \lambda)$  in period  $t$ . An increase in  $A_t$  implies an improvement in the matching efficiency — the main focus of this paper. An agent regains credit access with probability  $E_{\lambda'|\lambda} p_{t+1}(b', \lambda')$ , given the current productivity vector  $\lambda$  and choice  $b'$ .

Let  $\Omega_t$  be the distribution of agents over  $(b, \lambda, i)$  in period  $t$ . Given that the matching efficiency increases overtime, the distribution of agents  $\Omega_t$  also changes overtime. Therefore, prices, value functions, and policy functions are a function of time. Time has been suppressed from policy functions for ease of exposition.

### 1.3.1 Consumer problem

Let  $V_t^R(b, \lambda)$  be the value of repayment and  $V_t^D(\lambda)$  be the value of default in period  $t$ . Given an idiosyncratic state  $(b, \lambda)$ , the value of credit access  $V_t^C(b, \lambda)$  is the value of repayment if the agent cannot file for bankruptcy; otherwise, it is the maximum of the value of repayment and the value of default. The value of credit access is formalized by the following function:

---

<sup>5</sup> This assumption is the same as in Chatterjee, Corbae, Nakajima, and Rios-Rull (2006). It simplifies the model. However, the results are not sensitive to this assumption.

$$V_t^C(b, \boldsymbol{\lambda}) = \begin{cases} V_t^R(b, \boldsymbol{\lambda}) & \text{if } E_{\boldsymbol{\lambda}'|\boldsymbol{\lambda}}(\nu'_{\boldsymbol{\lambda}'})w_{t+1} - ew_{t+1} > 0.2b \\ \max[V_t^R(b, \boldsymbol{\lambda}), V_t^D(\boldsymbol{\lambda})] & \text{otherwise.} \end{cases}$$

The value of repayment is given by the following function:

$$\begin{aligned} V_t^R(b, \boldsymbol{\lambda}) &= \max_{c, b'} U(c) + \beta E_{\boldsymbol{\lambda}'|\boldsymbol{\lambda}} V_{t+1}^C(b', \boldsymbol{\lambda}') \\ \text{s.t.} \\ c + b &= w_t \nu_{\boldsymbol{\lambda}} + q_t(b', \boldsymbol{\lambda}) b' \\ c &\geq 0 \\ b' &\leq \bar{b}_t(\boldsymbol{\lambda}). \end{aligned}$$

In the above problem, given the wage rate and the bond price schedule, the agent chooses consumption and borrowing (or saving) to maximize the current-period utility of consumption and the discounted expected value of credit access. The constraints are the budget constraint, the non-negativity of consumption, and the borrowing constraint. The bond price schedule  $q_t(b', \boldsymbol{\lambda})$  is a function of borrowing and the current vector of productivity components. The model also assumes an exogenous price floor  $\underline{q}$  for issuing debt, which maps into a borrowing constraint  $\bar{b}_t(\boldsymbol{\lambda})$ .<sup>6</sup>

The value of default is given by the following function:

$$V_t^D(\boldsymbol{\lambda}) = U(w_t \nu_{\boldsymbol{\lambda}}) + \beta E_{\boldsymbol{\lambda}'|\boldsymbol{\lambda}} [p_{t+1}(0, \boldsymbol{\lambda}') V_{t+1}^C(0, \boldsymbol{\lambda}') + (1 - p_{t+1}(0, \boldsymbol{\lambda}')) V_{t+1}^N(0, \boldsymbol{\lambda}')].$$

The above function states that the agent who defaults consumes the labor earnings ( $c = w_t \lambda$ ); does not save or borrow ( $b' = 0$ ); and will regain access to credit with probability  $E_{\boldsymbol{\lambda}'|\boldsymbol{\lambda}} p_{t+1}(0, \boldsymbol{\lambda}')$  in the next period.

---

<sup>6</sup> The price floor ensures that the implications for spreads in the cross-section are not biased by a small mass of agents borrowing at extremely high spreads. The derivation of the borrowing constraint is discussed in section A.2 of the appendix.

The value of not having credit access is given by the following function:

$$\begin{aligned}
V_t^N(b, \boldsymbol{\lambda}) &= \max_{c, b'} U(c) + \beta E_{\boldsymbol{\lambda}'|\boldsymbol{\lambda}}[p_{t+1}(b', \boldsymbol{\lambda}')V_{t+1}^C(b', \boldsymbol{\lambda}') + (1 - p_{t+1}(b', \boldsymbol{\lambda}'))V_{t+1}^N(b', \boldsymbol{\lambda}')] \\
&\text{s.t.} \\
&c + b = w_t \nu_{\boldsymbol{\lambda}} + q_t(b', \boldsymbol{\lambda})b' \\
&c \geq 0 \\
&b' \leq 0.
\end{aligned}$$

In the above problem, given the wage rate, the bond price schedule, and the probability of credit access across all submarkets, the agent chooses consumption and saving to maximize utility. The constraints are the budget constraint, the non-negativity of consumption, and the no borrowing constraint. Note that the agent regains access to credit with probability  $E_{\boldsymbol{\lambda}'|\boldsymbol{\lambda}}p_{t+1}(b', \boldsymbol{\lambda}')$  in the subsequent period given current productivity vector  $\boldsymbol{\lambda}$  and choice  $b'$ .

### 1.3.2 The credit market

Given a search friction between borrowers and lenders, lenders incur a fixed cost  $\kappa$  to send one credit offer to a submarket  $(b, \boldsymbol{\lambda})$ . Given the lender's probability of a match, the zero profit condition for sending credit offers is as follows:

$$\kappa = A_t \frac{M(u_t(b, \boldsymbol{\lambda}), v_t(b, \boldsymbol{\lambda}))}{v_t(b, \boldsymbol{\lambda})} \Pi_t(b, \boldsymbol{\lambda}) \quad \forall (b, \boldsymbol{\lambda}) \in [B \times \boldsymbol{\Lambda}].$$

The zero profit condition equates the cost of sending a credit offer to the expected profits in a submarket  $(b, \boldsymbol{\lambda})$ . Function  $\Pi_t(b, \boldsymbol{\lambda})$  represents the present value of discounted flows for the lender.

$$\Pi_t(b, \boldsymbol{\lambda}) = \left[ -q_t(b'(b, \boldsymbol{\lambda}), \boldsymbol{\lambda})b'(b, \boldsymbol{\lambda}) + \frac{E_{\boldsymbol{\lambda}'|\boldsymbol{\lambda}}[1 - d(b'(b, \boldsymbol{\lambda}), \boldsymbol{\lambda}')] [b'(b, \boldsymbol{\lambda}) + \Pi_{t+1}(b'(b, \boldsymbol{\lambda}), \boldsymbol{\lambda}')] ]}{1 + r_{t+1}} \right].$$

The flows are calculated recursively given the policy function for borrowing, policy function for the default decision, and the bond price schedule:  $\{b'(b, \boldsymbol{\lambda}), d(b, \boldsymbol{\lambda})\}_{(b, \boldsymbol{\lambda}) \in [B \times \boldsymbol{\Lambda}]}$  and  $q_t(b', \boldsymbol{\lambda})_{(b', \boldsymbol{\lambda}) \in [B \times \boldsymbol{\Lambda}]}$ .  $-q_t(b'(b, \boldsymbol{\lambda}), \boldsymbol{\lambda})b'(b, \boldsymbol{\lambda})$  are the flows out if the agent borrows

and the flows in if the agent saves. The rest of the right hand side in the above expression is the discounted value of expected flows given choice  $b'(b, \boldsymbol{\lambda})$ , current productivity vector  $\boldsymbol{\lambda}$ , and the default policy function.

### 1.3.3 Bond price schedule

Following Livshits, MacGee, and Tertilt (2010) and Herkenhoff (2015), this paper assumes an exogenous lending fee  $\tau$ . That is, the borrowers have all the bargaining power, guarantee a proportional lending fee  $\tau$  when issuing debt, and make take-it-or-leave-it offers. This leads to the following bond price schedule:

$$q_t(b', \boldsymbol{\lambda}) = \begin{cases} \frac{1}{1 + r_{t+1}} & \text{if } b' \leq 0 \\ \frac{E_{\boldsymbol{\lambda}'|\boldsymbol{\lambda}}[1 - d(b', \boldsymbol{\lambda}')] ]}{1 + r_{t+1} + \tau} & \text{if } 0 < b' \leq \bar{b}_t(\boldsymbol{\lambda}). \end{cases}$$

Note that without a lending fee  $\tau$ , upon a match, lenders will make zero profits and have no incentive to send credit offers. An alternative is to give the lender some bargaining power and let the borrower and lender bargain over the surpluses. However, the assumption of take-it-or-leave-it offers makes the model tractable.

If the agent saves, then the bond price is discounted by the risk-free rate for the next period. If the agent borrows, then in addition to the risk-free rate, the bond price is discounted by the lending fee and also the probability of default.

Given the bond price schedule, the lender makes a profit only when the agent issues debt. Therefore, the flows from a submarket  $\Pi_t(b, \boldsymbol{\lambda})$  depends on how much an agent borrows and the probability of repayment. Finally, aggregate capital  $K_{t+1}$  is pinned down using the budget constraint of the financial intermediary. It is presented in section A.2 of the appendix. Next, we turn to the definition of a competitive equilibrium.



### 1.3.4 Competitive equilibrium

Given a sequence of matching efficiencies  $\{A_t\}_{t=1}^\infty$ , initial conditions for capital  $K_0$  and the distribution of agents  $\Omega_0$ , a competitive equilibrium is a sequence of individual value and policy functions for the households  $\{V_t, c_t, b'_t, d_t : \mathbb{R} \times \mathbf{\Lambda} \times I \rightarrow \mathbb{R}\}_{t=1}^\infty$ , a sequence of probabilities for credit access  $\{p_t : \mathbb{R} \times \mathbf{\Lambda} \rightarrow \mathbb{R}\}_{t=1}^\infty$ , a sequence of production plans for the firm  $\{K_t, L_t\}_{t=1}^\infty$ , a sequence of prices  $\{r_t, w_t\}_{t=1}^\infty$ , a sequence of bond price schedules  $\{q_t : \mathbb{R} \times \mathbf{\Lambda} \times I \rightarrow \mathbb{R}\}_{t=1}^\infty$ , and a sequence of measures  $\{\Omega_t\}_{t=1}^\infty$  such that, for all  $t$ , the following hold:

- Given prices, consumers solve their respective problems;
- Zero profit condition for credit offers;
- Prices  $r_t$  and  $w_t$  satisfy

$$r_t = \alpha \theta^{1-\alpha} (K_t/L_t)^{\alpha-1} - \delta,$$

$$w_t = (1 - \alpha) \theta^{1-\alpha} (K_t/L_t)^\alpha;$$

- Law of motion for  $\Omega_t$  is consistent with the policy functions and the exogenous productivity process; and
- Markets clear;

$$L_t = \int \nu_{\mathbf{\Lambda}} \Omega_t(db \times d\mathbf{\Lambda} \times di)$$

$$Y_t = \int c_t(b, \mathbf{\Lambda}, i) \Omega_t(db \times d\mathbf{\Lambda} \times di) + K_{t+1} - (1 - \delta)K_t.$$

## 1.4 Functional forms and parameterization

The utility function is standard CRRA given by,

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where  $\sigma$  is relative risk aversion. The matching function between borrowers and lenders is Cobb-Douglas, given by

$$M(u, v) = u^{\alpha_c} v^{1-\alpha_c}.$$

Production function is also Cobb-Douglas, given by

$$Y = K^\alpha (\theta L)^{1-\alpha},$$

where  $\theta$  dictates the scale of the economy.<sup>7</sup>

Table 1.1: **Parameters specified before calibration**

Parameter	Description		Value
$\alpha$	Capital share	Standard	0.3600
$\sigma$	Risk aversion		2.0000
$\rho_\epsilon$	Persistence	Güvenen, Ozkan, and Song (2014)	0.9530
$\sigma_\epsilon^2$	Variance of persistent component		0.0595
$\sigma_z^2$	Variance of transitory component		0.0299
$\sigma_{\eta_0}^2$	Variance of permanent component	Storesletten, Telmer, and Yaron (2004)	0.2105
$\underline{q}$	Bond price floor	Average penalty rate = 28.45 percent	0.7142
$\kappa$	Posting cost	Herkenhoff (2015)	1e-5
$\tau$	Lending fee	Base fee	0.0300
$e$	Allowable expense	Bankruptcy expense formula (\$17,443)	0.3594

Table 1.1 shows the parameters specified before calibration. The capital share of 0.36 and risk aversion of 2 are standard parameters taken from the macro literature. Recall that idiosyncratic productivity vector  $\boldsymbol{\lambda}$  is comprised of a persistent, transitory, and permanent component. The persistent component  $\gamma$  follows an  $AR(1)$  process given by  $\gamma' = \rho_\epsilon \gamma + \epsilon'$ . The draws for  $\epsilon'$ ,  $z$ , and  $\eta_0$  are iid normal with variances,  $\sigma_\epsilon^2$ ,  $\sigma_z^2$ , and  $\sigma_{\eta_0}^2$ , respectively. The productivity process is parameterized using estimates from Güvenen, Ozkan, and Song (2014) and Storesletten, Telmer, and Yaron (2004). The process is discretized using Rouwenhorst (1995) with eleven grid points for the persistent component and two grid points each for the transitory and permanent components.

The bond price floor  $\underline{q}$  is set such that the maximum lending rate is 40 percent. This is higher than the observed average penalty rate on credit cards of 28.45 percent, taken from the 2014 Penalty Rate survey conducted by CreditCards.com. Recall that the reason for the bond price floor is to ensure the analysis is not biased by a small mass of agents borrowing at extremely high spreads.<sup>8</sup> The cost of sending a credit offer  $\kappa$  is normalized to an arbitrarily small number, following Herkenhoff (2015). It is jointly determined with matching efficiency  $A$ , which will be calibrated.

<sup>7</sup> Given a Cobb-Douglas matching function, the probability of access to credit is not bounded above. However, the probability never exceeds one in all the quantitative exercises. There are computational gains to this assumption because it leads to an analytical solution for the probability of credit access.

<sup>8</sup> Results are not sensitive even if the maximum lending rate is increased to 70 percent.

The lending fee  $\tau$  of 0.03 is the base fee charged over the risk-free rate on prime customers, taken from Discover.com.<sup>9</sup> The fraction of allowable expense  $e$  is pinned down using expense guidelines for consumer bankruptcy given by the IRS Collection Financial Standards. The IRS has standards for four broad groups of expenses: local housing utilities, local transportation, national living expense, and national health expense. Using these guidelines, expenses are calculated for an average household of 2.59 members. Then, the expenses are converted to those of a working-age person and normalized by GDP per working-age person. The final calculation amounts to \$17,443 of deductible expenses.

Table 1.2: **Matching efficiency: parameters calibrated**

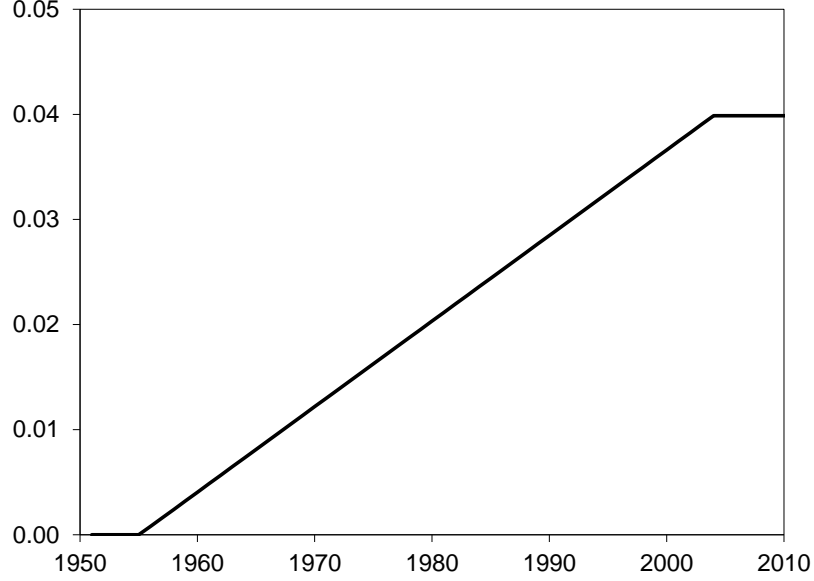
Parameter	Target	Value
$A_{1970}$	Matching efficiency 1970 <i>Unsecured credit access</i> $_{1970} = 18.46$ percent	0.0122
$A_{2004}$	Matching efficiency 2004 <i>Unsecured credit access</i> $_{2004} = 71.47$ percent	0.0399
$\beta$	Discount rate <i>Annual capital to output</i> $_{1970} = 3$	0.9187
$\theta$	Labor productivity <i>GDP per working age person</i> $_{1970} = 1$	0.5390
$\delta$	Depreciation rate <i>Interest rate</i> $_{1970} = 4.00$ percent	0.0806
$\alpha_c$	Borrower share <i>Unsecured credit to GDP</i> $_{1970} = 0.40$ percent	0.6594

The 1970 level of matching efficiency  $A_{1970}$  is calibrated to match the population with unsecured credit access in 1970. The 2004 level of matching efficiency  $A_{2004}$  is calibrated to match the population with unsecured credit access in 2004. Given  $A_{1970}$  and  $A_{2004}$ , a linear trend is assumed from 1951 to 2004.<sup>10</sup> From 2005 and after, the matching efficiency is kept constant at its 2004 level. Figure 1.4 shows the exogenous deterministic path for matching efficiency.

The four remaining parameters are the discount rate  $\beta$ , the aggregate labor productivity  $\theta$ , the depreciation rate  $\delta$ , and the borrower share in the matching function  $\alpha_c$ . These parameters are calibrated to match the following moments in 1970: annual capital to output ratio of 3; GDP per working-age person of 1; interest rate equal to 4 percent; and unsecured credit to GDP equal to 0.40 percent.

<sup>9</sup> Industry standard is to charge 3 percentage points over the risk-free rate on prime (safest) customers.

<sup>10</sup> Given a zero lower bound, matching efficiency is set to zero for years where the linear trend is negative.

Figure 1.4: **Exogenous deterministic path for matching efficiency A**

## 1.5 Results: increase in matching efficiency A

Table 1.3 shows that an increase in the matching efficiency accounts for more than 60 percent of the rise in unsecured credit. Table 1.4 shows the results for consumer bankruptcies. Neither the level nor the rise of consumer bankruptcies was calibrated, but matching efficiency does well in explaining both. The bankruptcy rate increases from 0.09 to 0.57 in the data and 0.19 to 0.60 in the model. Table 1.5 shows the percentage point change for the charge-off rate, the average spread, and the standard deviation of spreads. It shows an increase in the charge-off rate, 2.98 in the data and 1.86 in the model. The spread between the lending rate and the risk-free rate is calculated as follows:

$$spread_t = \overbrace{\frac{1}{q_t(b', \lambda)}}^{\text{lending rate}} - 1 - r_t$$

The average and standard deviation of spreads are computed in the simulation for the cross-section of agents who borrow in the given period. As Table 1.5 indicates, an increase in matching efficiency leads to a small change in the average spread, 0.37 in the data and 0.55 in the model. However, it quantitatively underestimates the increase in the standard deviation, 2.84 in the data and 0.21 in the model.

Table 1.3: **Unsecured credit** (percent of GDP)

	Average 1970-73	Average 2001-04	percentage point change
Data	0.62	6.59	5.97
Matching $\uparrow$ (benchmark)	0.97	4.95	3.98

Table 1.4: **Consumer bankruptcies** (percent of working-age population)

	Average 1970-73	Average 2001-04	percentage point change
Data	0.09	0.57	0.48
Matching $\uparrow$ (benchmark)	0.19	0.60	0.41

Table 1.5: **Implications for other measures** (percentage point change)

	Charge-off <sup>11</sup>	Spread	
		Average	Std. dev.
Data	2.98	0.37	2.84
Matching $\uparrow$ (benchmark)	1.86	0.55	0.21

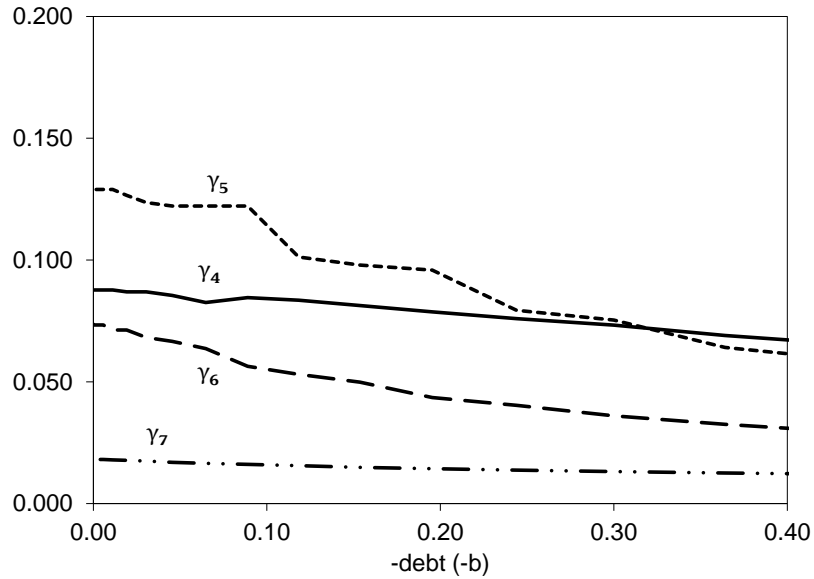
### 1.5.1 Discussion

Let us first discuss as to why an increase in the matching efficiency of directed search quantitatively accounts for the rise in credit and bankruptcies. The probability of credit access in a  $(b, \boldsymbol{\lambda})$  submarket is derived from the zero profit condition, as follows:

$$p_t(b, \boldsymbol{\lambda}) = A_t^{1/\alpha_c} \left[ \frac{\Pi_t(b, \boldsymbol{\lambda})}{\kappa} \right]^{(1-\alpha_c)/\alpha_c}.$$

Notice that the probability of credit access is a function of the level of matching efficiency, the profitability of the respective submarket, and the cost of sending a credit offer. Therefore, an increase in the matching efficiency  $A_t$  increases the probability of credit access across all submarkets, especially those that are the most profitable. Given that the lenders make profits when the agents borrow, the most profitable submarkets are the ones where agents are most likely to sustain high levels of debt. Figure 1.5 presents an example of the profit function  $\Pi(b, \lambda)$  in submarkets with different persistent productivities and asset levels (-debt).<sup>12</sup>

Figure 1.5: **Matching efficiency: example of profits by persistent productivity (year=1974)**



With respect to assets (-debt), we see that agents with low assets are more profitable. This is because they are more likely to borrow. With respect to productivities, agents with medium productivity shocks ( $\gamma_5$ ) are the most profitable because they are

<sup>12</sup> There are eleven grid points for the persistent productivity. The figure shows profits for the productivities which are the most profitable.

likely to borrow and sustain higher levels of debt. Agents with the highest shocks are less profitable because they are less likely to borrow. Agents with lowest shocks are also less profitable because they are more likely to default.

Figure 1.6: **Matching efficiency: population with credit access by persistent productivity (percent)**

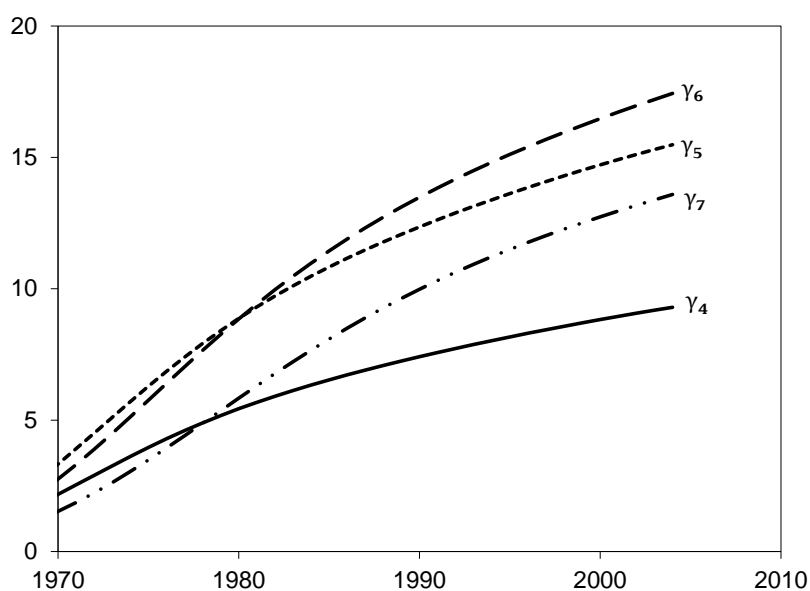
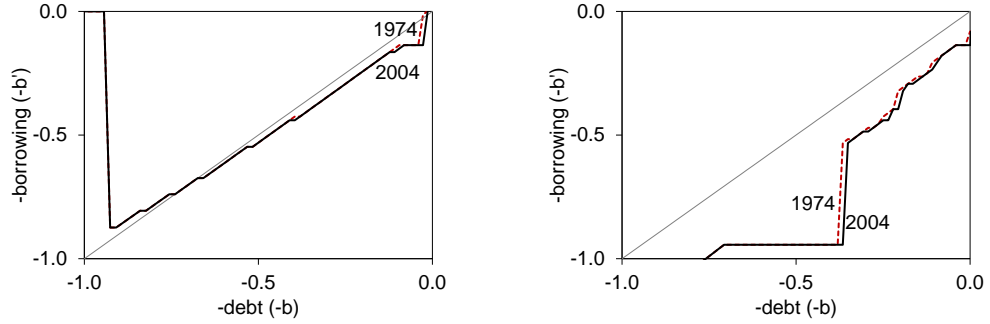


Figure 1.6 shows the increase in access to credit by different persistent productivities. It indicates that an increase in the matching efficiency increases access to credit, especially for agents who are more likely to borrow (more profitable). This generates the quantitative rise in both unsecured credit and consumer bankruptcies for an increase in the matching efficiency.

It is also important to note the role of the legal feature in enabling the model to generate a rise in both credit and bankruptcies for all the explanations studied in this paper. Let us first consider an environment without the legal feature. In such an environment, given the unsecured nature of credit, although the agent would like to borrow

in medium states to increase consumption, the agent cannot commit to repay in the next period. Given the high default probability, the agent will be faced with a high cost of borrowing (lower bond price). This dampens the rise in unsecured credit and consumer bankruptcies. In an environment with the legal feature, the agent cannot file when expected earnings is high. That is, the agent can commit to repay in high states. Therefore the agent in medium states can issue debt at a lower cost (higher bond price) compared to an economy without the legal feature. This generates the quantitative rise in both unsecured credit and consumer bankruptcies for all the explanations studied in this paper.

Figure 1.7: **Matching efficiency: examples of policy function for borrowing**

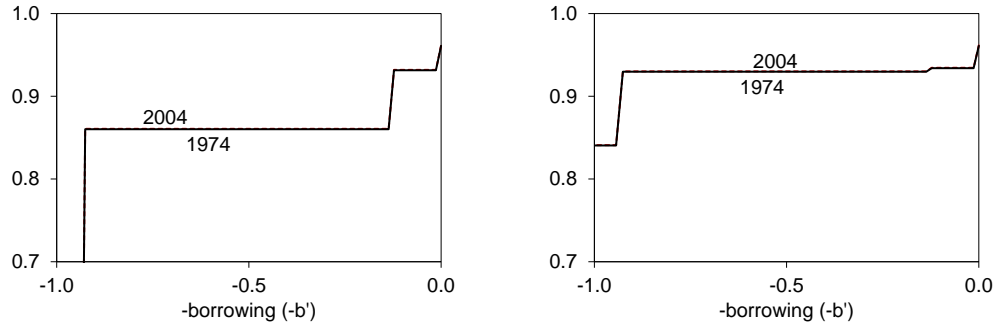


An increase in the matching efficiency is also consistent with additional evidence on the charge-off rate and the (cross-sectional) average spread. The charge-off rate increases because we are observing the transition path from an initial steady state with a matching efficiency of zero. That is, the unsecured credit market did not exist in the initial steady state and we are observing the economy transitioning from an equilibrium with zero unsecured credit to an equilibrium with positive credit. With persistent shocks, some agents accumulate debt. This can be seen in Figure 1.7 which displays the policy function for borrowing in 1974 and 2004 for two productivities. It shows that agents accumulate debt for these given productivity levels in both 1974 and 2004. Given the legal feature, agents are more likely to default with more debt. That is, with



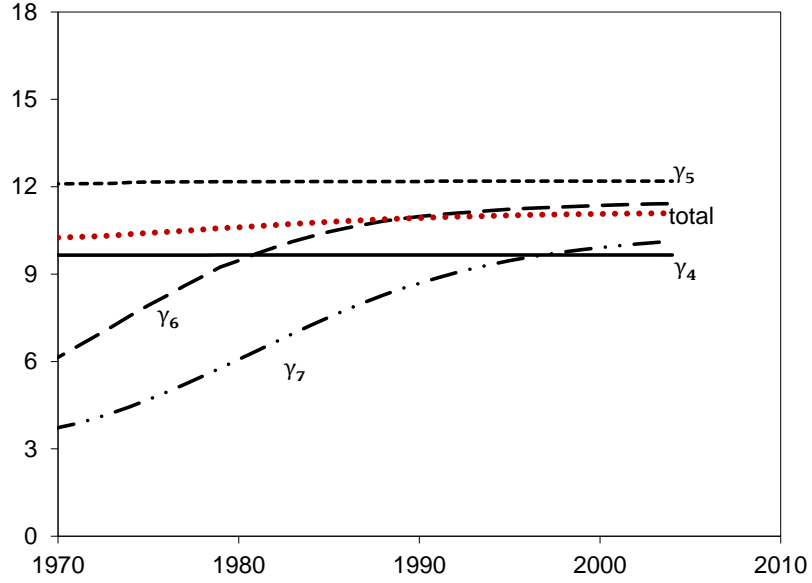
a higher level of debt, the agent is allowed to default with higher productivity realizations. The higher default probability is reflected in the bond price schedule presented in Figure 1.8. It shows that the bond price schedule decreases for higher debt levels (again in both 1974 and 2004).

Figure 1.8: **Matching efficiency: examples of bond price schedule**



An increase in the matching efficiency leads to small change in the (cross-sectional) average spread for the following reason: recall that access disproportionately increases for agents who are the most profitable. These are agents with medium shocks and low assets. An increase in the mass of agents from the same pool has little impact on the average. This can be seen in Figure 1.9 which presents the average spread by persistent productivities and the average for the whole cross-section of debtors. An increase in the mass of agents with productivities  $\gamma_4$  and  $\gamma_5$  has little impact on the average. An increase in the matching efficiency also increases access for agents who are less profitable. In this case, these agents are relatively safer ( $\gamma_6$  and  $\gamma_7$ ). The fact that they are relatively safer compared to the current pool of debtors puts downward pressure on the total average. However, as they accumulate debt, they become more risky which puts upward pressure on the total average. This leads to a small change in the average spread, consistent with the data.

Figure 1.9: **Matching efficiency: average spread by persistent productivity (percentage points)**



### 1.5.2 Endogenous vs exogenous probability of credit access

The previous section argued that improved matching quantitatively accounts for the rise in unsecured credit and consumer bankruptcies because it increases access to agents who are more likely to borrow. Therefore, this section considers an alternative set up where the probability of credit access is exogenous and uniform across all agents. Suppose the probability of credit access in period  $t$  is a constant, as follows:

$$p_t(b, \lambda) = p_t.$$

The model is re-calibrated to match the rise in credit access. Table 1.6 compares the results for the rise in unsecured credit and consumer bankruptcies. It indicates that the endogenous probability of credit access in improved matching is important to quantitatively account for the rise in credit and bankruptcies.

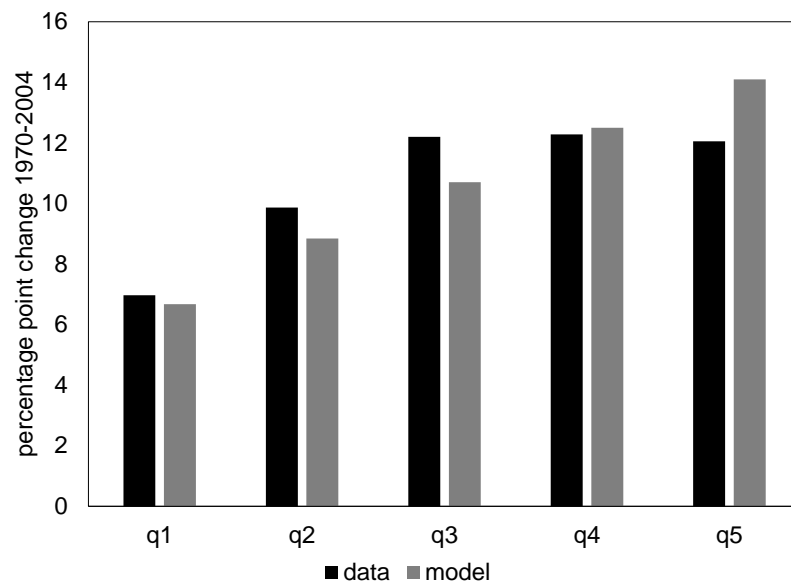
Table 1.6: **Matching vs exogenous probability of credit access** (percentage point change)

	Unsecured credit	Consumer bankruptcies
Data	5.97	0.48
Matching $\uparrow$ (benchmark)	3.98	0.41
Exogenous probability of credit access	1.36	0.18

### 1.5.3 Additional evidence for mechanism

Figure 1.10 compares the change in access to credit by income quintile between the model and data. Access to credit has increased more for the higher income quintiles. The model is consistent with this observed behavior.

Figure 1.10: **Access to credit by income quintile**



## 1.6 Alternative explanations

This section explores the following alternative explanations for the rise in unsecured credit and consumer bankruptcies: (1) a decrease in the cost of bankruptcy; (2) a decrease in the lending fee; and (3) an increase in the lender's information about the borrower's characteristics.

### 1.6.1 Decrease in the cost of bankruptcy (stigma)

In the benchmark model, the cost of bankruptcy was loss of credit access and inability to save during the period of default. The benchmark model is now extended to include a utility cost  $\chi_t$  faced by agents during the period of default, referred to as *stigma* by Athreya (2004) and Livshits, MacGee, and Tertilt (2010). It is a measure of the non-pecuniary cost associated with filing for bankruptcy. Now the value of default is given by the following function:

$$V_t^D(\lambda) = U(w_t\nu_\lambda) - \chi_t + \beta E_{\lambda'|\lambda}[p_{t+1}(0, \lambda')V_{t+1}^C(0, \lambda') + (1 - p_{t+1}(0, \lambda'))V_{t+1}^N(0, \lambda')].$$

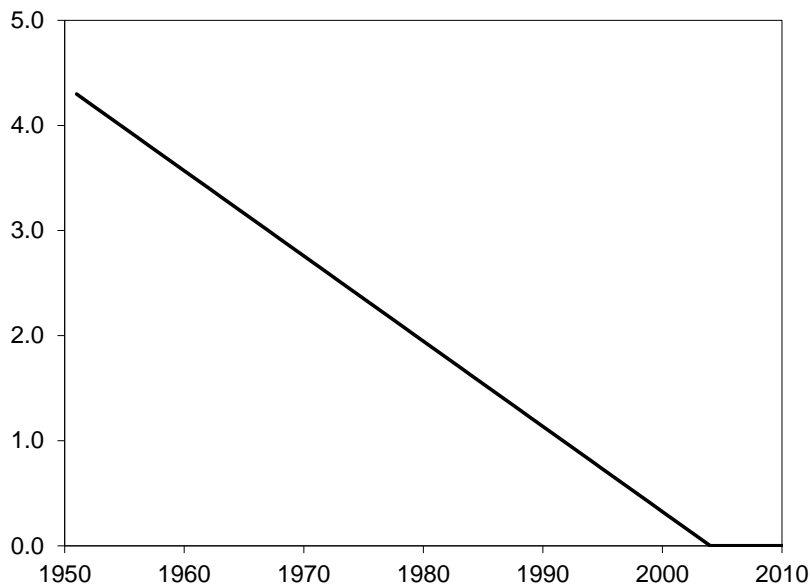
That is, the agent who defaults consumes the labor earnings ( $c = w_t\lambda$ ); does not save or borrow ( $b' = 0$ ); regains access to credit with probability  $E_{\lambda'|\lambda}p_{t+1}(0, \lambda')$ ; and faces an additional utility cost (stigma)  $\chi_t$  of filing for bankruptcy. The rest of the model is unchanged.<sup>13</sup>

For this exercise, there are seven parameters calibrated to match seven moments in the U.S. economy. Table A.2 presents the calibration numbers for all the alternative explanations in a separate section (Section A.5). The 1970 level of stigma  $\chi_{1970}$  is calibrated to match the number of bankruptcies per working-age person in 1970. The 2004 level of stigma  $\chi_{2004}$  is set to zero to get the maximum increase in consumer bankruptcies. Given  $\chi_{1970}$  and  $\chi_{2004}$ , a linear trend is assumed for the period from 1951 to 2004. From 2005 and after, stigma is zero at its 2004 level. Figure 1.11 shows the exogenous deterministic path for stigma. In this exercise, the matching efficiency is kept constant and is calibrated to match the population with unsecured credit access in 2004. The four

<sup>13</sup> In this model, agents face no stigma after the period of default. In Livshits, MacGee, and Tertilt (2010), agents face a utility cost for the whole period they cannot borrow. Results are not sensitive to this alternative setup because stigma is a utility cost that will be calibrated later. However, it simplifies the model.

remaining parameters are calibrated to match the same four moments as in Section 1.4.

Figure 1.11: **Exogenous deterministic path for stigma  $\chi$**



The results for all the alternative explanations are presented in Table 1.7, Table 1.8, and Table 1.9. Table 1.7 shows that a decrease in stigma explains almost 60 percent of the rise in unsecured credit. Table 1.8 shows that a decrease in stigma does well in explaining the rise in bankruptcies. It increases from 0.09 to 0.57 in the data compared to 0.11 to 0.51 in the model. As Table 1.9 shows, a decrease in stigma generates an increase in the charge-off rate, 2.98 in the data and 2.12 in the model. Therefore, a decrease in stigma does equally well compared to an increase in matching efficiency with respect to the rise in unsecured credit, consumer bankruptcies, and the charge-off rate. However, a decrease in stigma over-estimates the increase in the average spread, 0.37 in the data and 3.60 in the model; and leads to a decrease in the standard deviation of spreads, 2.84 in the data and -0.29 in the model. The rest of this section discusses the mechanism in more detail.

Table 1.7: **Unsecured credit** (percent of GDP)

	Average 1970-73	Average 2001-04	percentage point change
Data	0.62	6.59	5.97
Stigma ↓	0.38	3.83	3.44
Lending fee ↓	1.73	4.10	2.36
Stigma ↓ & lending fee ↓	0.23	3.34	3.11
Information ↑	0.22	3.40	3.18

Table 1.8: **Consumer bankruptcies** (percent of working-age population)

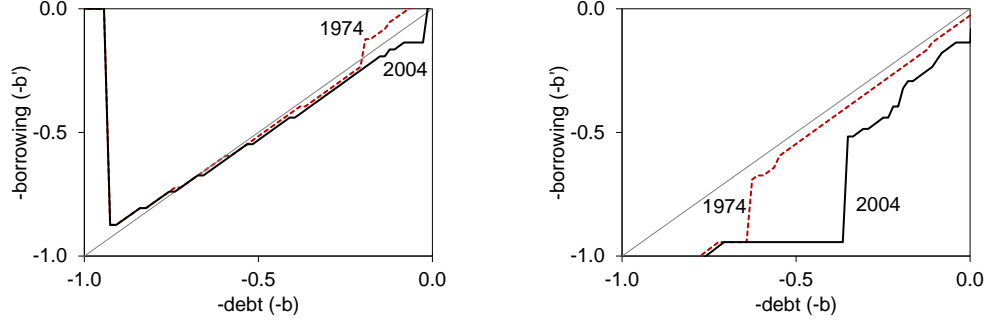
	Average 1970-73	Average 2001-04	percentage point change
Data	0.09	0.57	0.48
Stigma ↓	0.11	0.51	0.41
Lending fee ↓	0.35	0.54	0.19
Stigma ↓ & lending fee ↓	0.10	0.48	0.38
Information ↑	0.14	0.50	0.36

A decrease in stigma increases the value of default  $V_t^D(\lambda)$ . This, in turn, increases the agent's incentive to take on levels of debt associated with higher default probabilities. This change in behavior is presented in Figure 1.12, which compares the policy function for borrowing between 1974 and 2004 for two different productivity levels. In both cases, the agent in 2004 is willing to take on higher levels of debt compared to the agent in 1974 with the same productivity and debt levels. The increase in the value of default leads agents to borrow more and repay debt at a slower rate. This increases the mass of agents with debt, along with an increase in unsecured credit and consumer bankruptcies.

The charge-off rate increases because agents take on levels of debt with a higher default probability in 2004 compared to 1974. However, this also leads to a counterfactual

Table 1.9: **Implications for other measures** (percentage point change)

	Charge-off <sup>14</sup>	Spread	
		Average	Std. dev.
Data	2.98	0.37	2.84
Stigma ↓	2.12	3.60	-0.29
Lending fee ↓	0.28	-2.74	0.16
Stigma ↓ & lending fee ↓	1.22	-0.70	-0.13
Information ↑	1.75	-1.22	-0.87

Figure 1.12: **Stigma ( $\chi$ ): examples of policy functions for borrowing**

increase in the average spread. This can be seen in the following equation:

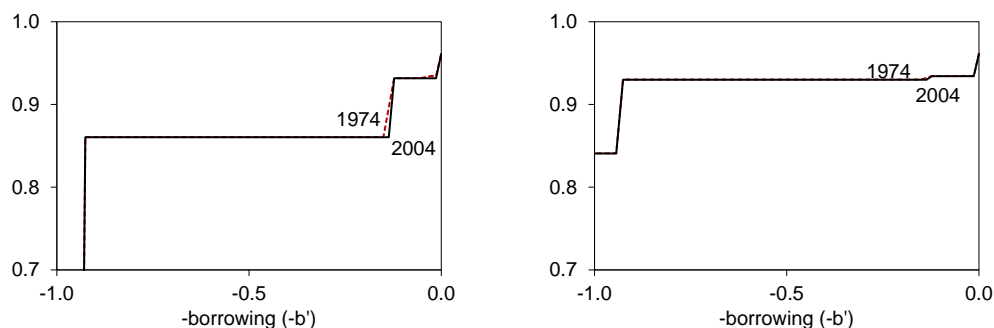
$$spread_t = \frac{1 + r_{t+1} + \tau}{E_{\mathbf{\lambda}'|\mathbf{\lambda}}[1 - d(b', \mathbf{\lambda}')] } - 1 - r_t.$$

In the above equation, the probability of default increases with a higher  $b'$  which increases the spread, and therefore, also the average. We observe a decrease in the standard deviation of spreads because a decrease in stigma disproportionately increases the mass of debtors with higher risk of default.

Note that the increase in the value of default also increases the probability of default holding constant the debt level. That is, it can change both the policy function for borrowing and the policy function for default. The latter decreases the bond price schedule and can reduce the incentive to borrow. If the second mechanism dominates, it dampens the rise or even decreases unsecured credit. This second mechanism dominates in Athreya (2004) and Livshits, MacGee, and Tertilt (2010). In this paper, however,

the second mechanism is dampened because of the legislative feature that ensures that agents cannot default if they can repay 20 percent of their debt in the next period. That is, in most states in which agents can choose to default, they were already choosing to do so even with high stigma in 1974. This can also be seen in Figure 1.13, where the decrease in bond price schedules between 1974 and 2004 is small. Therefore, the increase in the cost of borrowing is dampened.

Figure 1.13: **Stigma ( $\chi$ ): examples of bond price schedule**



### 1.6.2 Decrease in the lending fee

The benchmark model assumes a constant lending fee. However, Athreya (2004) shows that a decrease in the lending fee, which decreases the cost of borrowing, leads to an increase in unsecured credit and consumer bankruptcies. Livshits, MacGee, and Tertilt (2010) show that a decrease in the stigma and the lending fee leads to an increase in unsecured credit and consumer bankruptcies.<sup>15</sup> The decrease in the lending fee can be interpreted as a lower markup due to increased competition in the market for unsecured credit. Therefore, this section explores a decrease in the lending fee  $\tau$  and its implications for the charge-off rate and the (cross-sectional) average and dispersion of spreads.

The lending fee is estimated using a methodology that builds on Livshits, MacGee,

<sup>15</sup> The difference is that Athreya (2004) assumes debt is not priced based on an individual's default probability.



and Tertilt (2010). They assume a representative agent and estimate  $\tau$  from early 1980s to late 1990s. This paper estimates  $\tau$  from 1983 to 2010, but in an environment with heterogeneous agents.

For a given period, let the charge-off rate (total fraction of debt written off) be given by the following expression:

$$\Delta = \frac{\sum_s d_s b_s}{\sum_s b_s},$$

where  $d_s$  and  $b_s$  are the default probability and debt level, respectively, of individual  $s$ . Let  $r_s$  be the lending rate for individual  $s$ . Individual default probability  $d_s$  can be backed out as a function of  $r_s$ ,  $\tau$ , and  $r_f$  from the following equation for the bond price schedule:

$$q_s = \frac{1 - d_s}{1 + r_f + \tau} = \frac{1}{1 + r_s}.$$

Substituting  $d_s(r_s, \tau, r_f)$  into the equation for the charge-off rate, we get the following expression for  $\tau$ :

$$\tau = \frac{(1 - \Delta) \sum_s b_s}{\sum_s b_s / (1 + r_s)} - (1 + r_f).$$

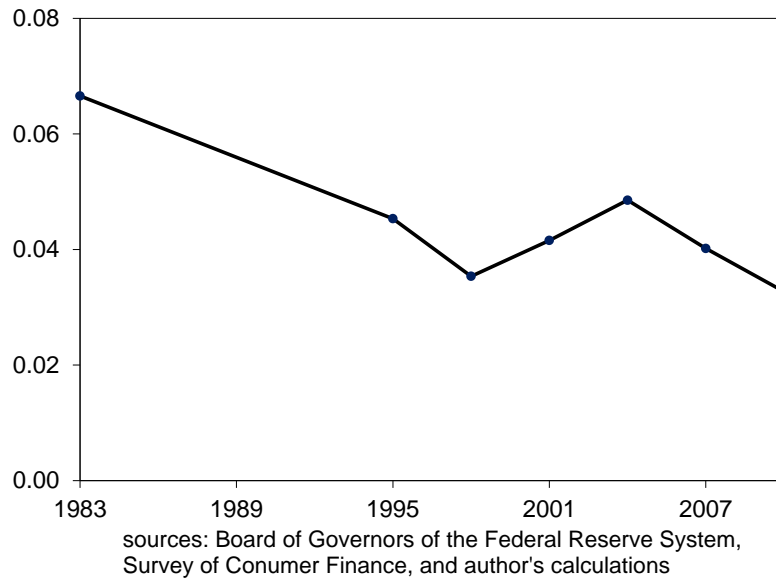
This expression is useful because we observe all the variables on the right hand side. Data on charge-off rates are available from Ausubel (1991) and the Board of Governors of the Federal Reserve System. Data on the risk-free rates are available from the Board of Governors of the Federal Reserve System. Data on individual debt and lending rates are available from the Survey of Consumer Finances.

Figure 1.14 presents the estimates for  $\tau$  from 1983 to 2010. It shows a decrease from 1983 until 1998. However, it also shows that  $\tau$  increased back to its 1995 level in 2004. Therefore, it is not clear whether the decrease in  $\tau$  is a permanent trend or a cyclical deviation.

The following exercise assumes  $\tau$  to be constant at its 1998 level from 1999 onwards. This ensures that this explanation generates the rise in unsecured credit and consumer bankruptcies; and then we can study implications for other measures for which annual data is available. For the period before 1983,  $\tau$  is assumed to be constant at its 1983 level. The deterministic transition path is presented in Figure 1.15. The calibrated parameters are presented in Table A.2.<sup>16</sup>

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<sup>16</sup> This exercise sets  $\alpha_c = 0.40$  because the initial steady state starts with a higher level of borrowing.

Figure 1.14: **Estimates for lending fee  $\tau$** 

The results are presented in Table 1.7, Table 1.8, and Table 1.9. The main point here is that a decrease in the lending fee also leads to an increase in unsecured credit and consumer bankruptcies, but it leads to a decrease in the average spread of -2.74 as compared to 0.37 in the data.

### **Both stigma and lending fee**

Recall that a decrease in the stigma over-estimates the increase in the average spread while a decrease in the lending fee leads to a decrease in the average spread. Therefore, the model is re-calibrated with both a decrease in the stigma and the lending fee to analyze if the effects cancel out and lead to a small change in the average spread, as observed in the data. The deterministic transition path for the lending fee is the same as before. The stigma is re-calibrated to match the same moments as in the previous section.

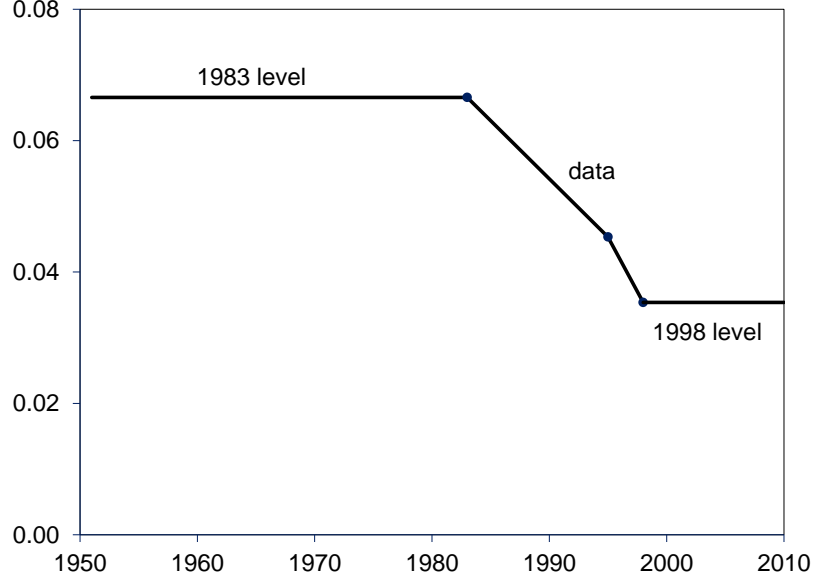
Figure 1.15: Exogenous deterministic path for lending fee  $\tau$ 

Table 1.7, Table 1.8, and Table 1.9 show the results for both a lower lending fee and a lower stigma. Now the model shows a small change in the average spread, consistent with the data. A similar result is pointed out by Livshits, MacGee, and Tertilt (2010) who look at average real interest rates instead of average spreads. Therefore the combination of a decrease in the stigma and the lending fee does equally well compared to an increase in the matching efficiency with respect to credit, bankruptcies, charge-off rate, and the average spread.

### 1.6.3 Increase in the lender's information

This section explores the role of an increase in the lender's information about the borrower's characteristics in explaining the rise in unsecured credit and consumer bankruptcies. It studies the effects of an increase in the lender's information about the borrower's persistent productivity component.

Recall that the idiosyncratic productivity vector is given by  $\boldsymbol{\lambda} = (\gamma, z, \eta_0)$ . This

section assumes that the lender observes the permanent component of productivity  $\eta_0$ . However, the lender does not observe the persistent component  $\gamma$  nor the transitory component  $z$ . Assuming that the lender observes the permanent component is reasonable because the permanent component refers to permanent heterogeneity due to differences in characteristics observed by the lender such as age and education. Assuming that the lender does not observe the persistent component leads to bond price schedules that will be a function of the lender's distribution of beliefs. Assuming that the lender does not observe the transitory component guarantees that the lender cannot back out the persistent component of productivity by observing labor earnings.

Suppose  $\gamma \in \{\gamma_1, \dots, \gamma_N\}$  is the true type, where the  $AR(1)$  process has been discretized with  $N$  grid points. Given  $\gamma$  and  $\phi_t$ , suppose that the distribution of beliefs of the lender is exogenously given by a convex combination of the ergodic probability distribution and the true probability distribution:

$$\psi_{belief}(\gamma_i, \gamma; \phi_t) = \phi_t \psi_{ergodic}(\gamma_i) + (1 - \phi_t) 1(\gamma_i = \gamma) \quad i \in \{1, 2, \dots, N\}$$

where  $\phi_t \in [0, 1]$  and  $\psi_{ergodic}$  is the ergodic probability distribution.  $\phi_t$  is the weight on the ergodic probability distribution and  $(1 - \phi_t)$  is the weight on true probability distribution. Note that  $\phi_t = 1$  is the full private information case, in which the lender's distribution of beliefs for any type  $\gamma$  is given by the ergodic distribution of the persistent productivity process.  $\phi_t = 0$  is the perfect information case, in which the lender perfectly observes the persistent productivity. Therefore, a decrease in  $\phi_t$  increases the lender's information.<sup>17</sup>

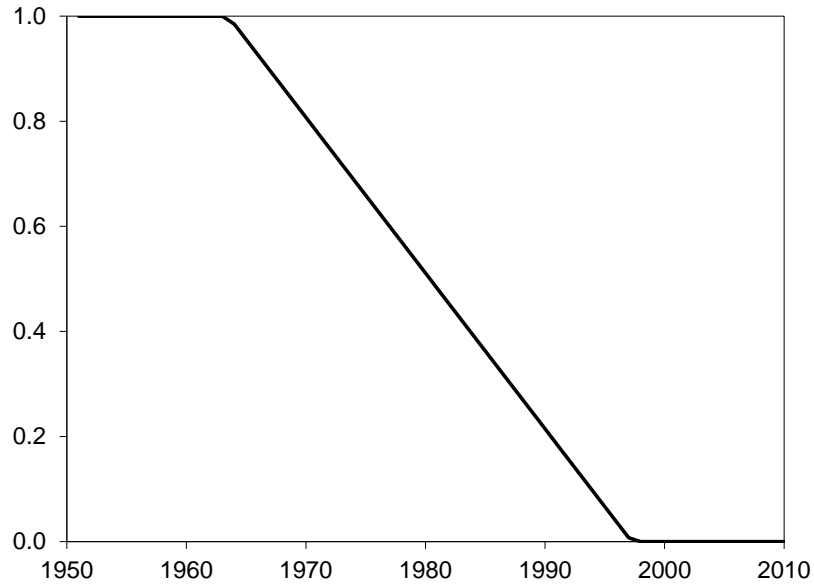
Given a true type  $\gamma$ , the bond price schedule is given by the following function where the default risk is calculated over the lender's distribution of beliefs:

$$q_t(b', \gamma, \eta_0; \phi_t) = \begin{cases} \frac{1}{1 + r_{t+1}} & \text{if } b' \leq 0 \\ \frac{\sum_i \psi_{belief}(\gamma_i, \gamma; \phi_t) E_{\gamma'|\gamma_i}[1 - d(b', \gamma', z', \eta_0)]}{1 + r_{t+1} + \tau} & \text{if } 0 < b' \leq \bar{b}_t(\gamma, \eta_0). \end{cases}$$

<sup>17</sup> An alternative is to allow lenders to learn about the productivity of the borrower through Bayesian updating. However, lack of learning gives the theory of an increase in the lender's information its best chance to explain the rise in credit and bankruptcies. It also makes the model tractable. Also note that the lender does not have to form a belief over the transitory component of productivity  $z$  to back out default probabilities for the next period.

All the other features are the same as in the benchmark model. For this exercise, we have seven parameters and seven moments. Again, the moments and respective parameters are presented in Table A.2. The 1970 level of information  $\phi_{1970}$  is calibrated to match the number of bankruptcies per working-age person in 1970. The 1997 level of information  $\phi_{1997}$  is calibrated to match the number of bankruptcies per working-age person in 2004. A linear trend is assumed for the period from 1951 to 1997.<sup>18</sup> From 1998 and after,  $\phi$  is kept constant at its lower bound of zero, as shown in Figure 1.16.

Figure 1.16: **Exogenous deterministic path for weight on false type  $\phi$**



Matching efficiency is kept constant and calibrated to match the population with unsecured credit access in 2004. The remaining parameters are calibrated to match the same moments as in Section 1.5.

Table 1.7, Table 1.8, and Table 1.9 show the results from the exercise. They

<sup>18</sup> A linear trend from 1970 to 1997 instead of 1970 to 2004 is assumed so that the exercise of an increase in the lender's information can actually match the increase in the number of bankruptcies per working-age person.

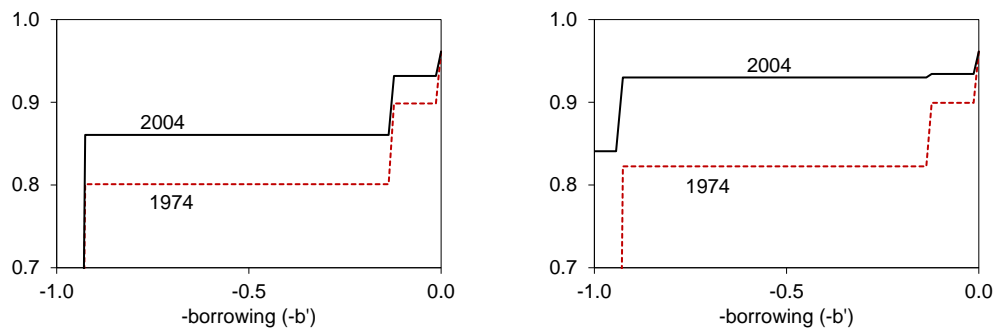
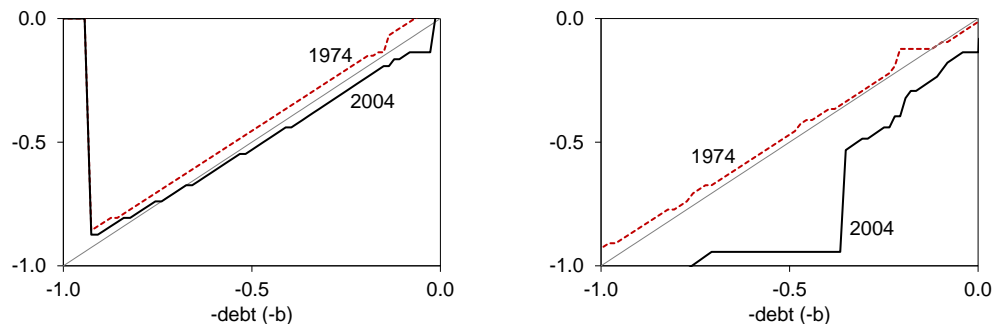
show an increase in unsecured credit, consumer bankruptcies, and the charge-off rate. Therefore, an increase in the lender's information does equally well compared to an increase in the matching efficiency with respect to credit, bankruptcies, and the charge-off rate. An increase in the lender's information, however, leads to a decrease in the average spread of -1.22 as compared to 0.37 in the data; it also leads to a decrease in the dispersion of spreads of -0.87 as compared to 2.84 in the data.

To understand the results, let us consider the perfect information case where bond price schedules reflect the true probability of default. With perfect information, the majority of agents who borrow, are those with medium shocks ( $\gamma_5$  and  $\gamma_6$ ) to their persistent component of earnings. These are agents who will repay if their earnings increase, but will default if their earnings decrease. In equilibrium, they borrow at an average spread of 11 percentage points. If the probability of default were zero, then they would borrow at a spread of 3 percentage points (the lending fee). Agents with lower shocks to the persistent component of earnings account for a smaller fraction of borrowers because they default even for low levels of debt. Agents with higher shocks to the persistent productivity save.

With private information, the agents with medium shocks to earnings are hurt by the lender's belief that they might be less productive, which in turn, leads to worse bond price schedules. The increase in the lender's information allows these agents to issue debt at higher prices (lower spreads). We see this in Figure 1.17, where agents are able to issue debt at higher prices (lower spreads) in 2004 compared to 1974. Figure 1.18 indicates that an agent in 2004 is willing to borrow more compared to an agent in 1974. This leads to an increase in unsecured credit and consumer bankruptcies.

It is easy to understand why an increase in the lender's information increases the charge-off rate. Agents take on higher levels of debt. They are more likely to default with higher levels of debt. This increases the charge-off rate.

However, an increase in the lender's information leads to a decrease in the average spread for the exact reason it leads to an increase in unsecured credit and consumer bankruptcies — agents with medium shocks to earnings are able to issue debt at lower spreads. There is also a decrease in the dispersion of spreads because of the disproportionate increase in these borrowers with medium shocks.

Figure 1.17: **Lender's information: examples of bond price schedule**Figure 1.18: **Lender's information: examples of policy function for borrowing**

## 1.7 Conclusion

This paper shows that improved matching between borrowers and lenders accounts for the rise in unsecured credit and consumer bankruptcies in the United States. This explanation is consistent with the observed behavior of measures such as the charge-off rate and the (cross-sectional) average spread. The following three alternative explanations also increase unsecured credit and consumer bankruptcies: (1) a decrease in the cost of bankruptcy; (2) a decrease in the lending fee; and (3) an increase in the lender's information about the borrower's characteristics. However, these explanations are not consistent with the observed behavior of the (cross-sectional) average spread. Therefore, this paper concludes that improved matching between borrowers and lenders has led to

the rise in unsecured credit and consumer bankruptcies in the United States.



## Chapter 2

# Trend and Cycle of Low-Skilled Manufacturing Employment

### 2.1 Introduction

Low-skilled manufacturing employment as a percentage of population (16 and over), decreased by 9.7 percentage points from 1967 to 2015 for the U.S. economy.<sup>1</sup>,<sup>2</sup> Most of this decrease is observed during recessions (71 percent). We can see this in Figure 2.1, where recessions are shaded in gray as per dates given by the National Bureau of Economic Research. The net decrease is 6.9 percentage points during recessions and 2.8 percentage points during non-recessions. This is puzzling because it suggests an interaction of trend and cycle. This leads to the question that this paper addresses: Why is 71 percent of the decrease in low-skilled manufacturing employment observed during recessions?

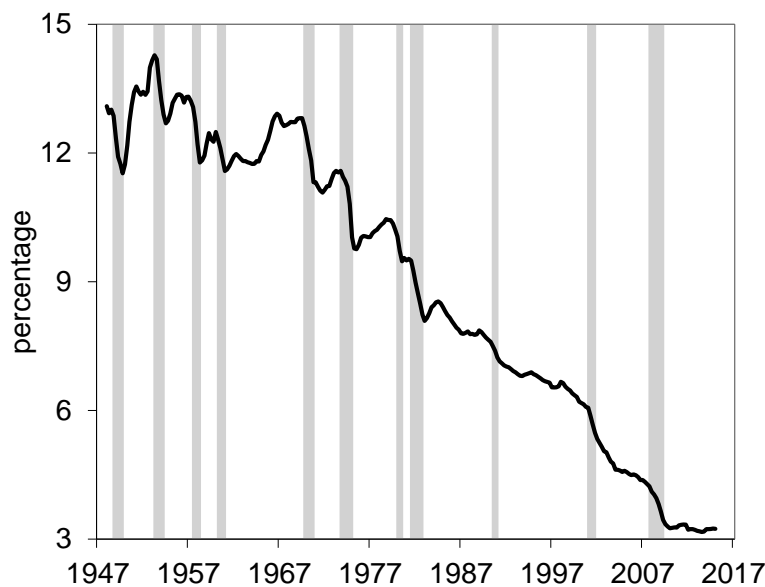
I argue that the above observation is a result of the interaction between investment-specific productivity and labor-augmenting productivity. Production is assumed to be a function of four inputs: capital structure, capital equipment, low-skilled manufacturing

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<sup>1</sup> A similar decrease has been documented for routine employment and low-skilled employment. The reason for the choice of low-skilled manufacturing employment is discussed in a later section. An employee with less than four years of college is classified as low-skilled.

<sup>2</sup> Construction of low-skilled manufacturing employment from 1948 to 2015 is discussed in the appendix.

Figure 2.1: **Low-skilled manufacturing employment to population (16 and over)**

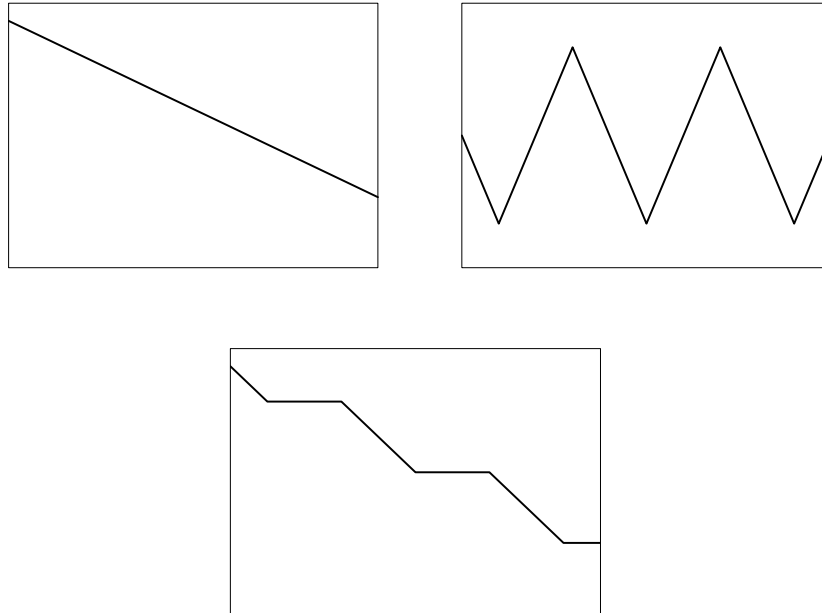


employment, and non-low-skilled manufacturing employment. Non-low-skilled manufacturing employment refers to employment that is high-skilled or employment not in manufacturing. The key assumption in the production function is stronger substitutability between capital equipment and low-skilled manufacturing employment compared to that of capital equipment and non-low-skilled manufacturing employment. Therefore, an increase in investment-specific productivity results in an increase in capital equipment, which leads to a decrease in low-skilled manufacturing employment. Fluctuations in labor-augmenting productivity (business cycles) lead to most of the decrease in the low-skilled manufacturing employment observed during recessions. This theory can be illustrated using an example, (Rogerson, 2014).

Suppose that a variable has a two-percent decreasing trend along with a two-percent cycle, as shown in Figure 2.2. When we look at the sum of trend and cycle, we observe a decrease in the variable during recessions, while the variable remains stable during

non-recessions. This is because the decrease in trend is offset by the post-recession recovery.

Figure 2.2: **Post-recession recovery offset by decreasing trend**



This example is the motivation of this paper. However, it is an extreme example as it requires the decrease in trend to be exactly offset by the cycle. As mentioned above, this paper hypothesizes that investment-specific productivity and labor-augmenting productivity contribute to most of the decrease in low-skilled manufacturing employment observed during recessions. Investment-specific productivity is computed as the inverse of the relative price of equipment to Gross Domestic Product (GDP), excluding equipment.<sup>3</sup> Labor-augmenting productivity is backed out from the production function by using data on both inputs and output.

I feed deterministic sequences of the relative price of equipment and labor-augmenting productivity into a frictionless general equilibrium model and compute the transition

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<sup>3</sup> The equivalence between investment-specific productivity and the relative price of equipment is proved in the appendix.

path from 1948 to 2015. Then, I compare the decrease in low-skilled manufacturing employment between the model and the data. I find that, in the model, the interaction of investment-specific productivity and labor-augmenting productivity leads to 52 percent of the decrease in low-skilled manufacturing employment observed during recessions (71 percent in the data).

The computed transition paths also have implications for jobless recoveries and movement in the labor share. In the data, we observe jobless recoveries in recessions post-1990.<sup>4</sup> The association of structural change with jobless recoveries has been controversial in the literature. My model generates most of the decrease in low-skilled manufacturing employment observed during recessions, but there are no jobless recoveries. In addition, the labor share in the data increased between 1948 and the 1960s and has decreased since then. Labor share in the model is qualitatively consistent with this pattern in the data.

## 2.2 Related Literature

My work builds on two aspects of Krusell, Ohanian, Rios-Rull, and Violante (2000). The first is in regards to the production function, in which they assume capital equipment complements high-skilled labor more than low-skilled labor. Instead of low-skilled, I use low-skilled manufacturing and instead of high-skilled, I use non-low-skilled manufacturing. Details of this choice is discussed in the next section. Second, Krusell, Ohanian, Rios-Rull, and Violante (2000) study how changes in the relative price of equipment affect the wage premium for high-skilled labor over low-skilled labor. I study how changes in the relative price of equipment and labor-augmenting productivity lead to most of the decrease in low-skilled manufacturing employment observed during recessions.

My work is also related to that of Jaimovich and Siu (2014) and Morin (2014). Jaimovich and Siu (2014) and Morin (2014) study the interaction between the decrease in routine employment (instead of low-skilled manufacturing employment) and jobless recoveries. Jaimovich and Siu (2014) propose a theory with search frictions, in which an increase in the productivity of non-routine labor leads to a transition of individuals

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<sup>4</sup> Jobless recovery refers to the slow and persistent recovery of employment after the recession.

from routine employment to non-routine employment. A recession amplifies the transition. However, individuals are also faced with a search friction which leads to jobless recoveries. Morin (2014) proposes a theory with hiring costs and decreasing relative equipment price leading to jobless recoveries. My work differs from theirs in two key aspects. First, I use low-skilled manufacturing employment instead of routine employment and document a decrease since the late 1960s instead of the late 1980s. Second, using a frictionless model, I find that most of the decrease in low-skilled manufacturing employment is observed during recessions. However, there are no jobless recoveries.

Lehn (2014) and Eden and Gaggl (2015) also compute transitions with the relative price of equipment and routine labor (instead of low-skilled manufacturing employment). However, they do not focus on the decrease observed during recessions or on jobless recoveries.

## 2.3 Data

### 2.3.1 Why low-skilled manufacturing employment?

The three standard classifications in the literature are based on low-skilled/high-skilled, manufacturing/non-manufacturing, and routine/non-routine. Following Krusell, Ohanian, Rios-Rull, and Violante (2000), an employee with at least four years of college is classified as high-skilled. Manufacturing/non-manufacturing is based on the 1990 industry classification. Routine/non-routine is a classification developed by Autor, Levy, and Murnane (2003) and Autor and Dorn (2013).

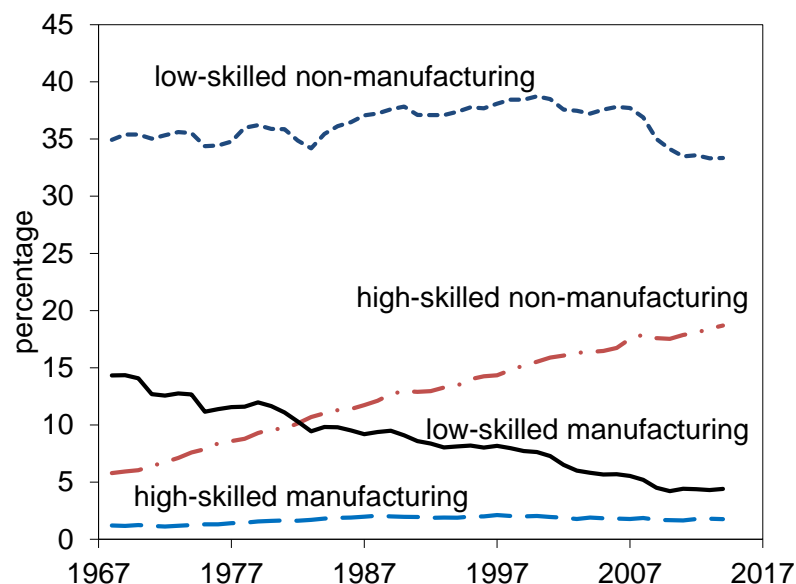
Jaimovich and Siu (2014), Morin (2014), and Eden and Gaggl (2015) use the routine/non-routine classification. Krusell, Ohanian, Rios-Rull, and Violante (2000) and Karabarbounis and Neiman (2014) use the low-skilled/high-skilled classification. Lehn (2014) uses routine, manual, and abstract classification. I use low-skilled manufacturing and non-low-skilled manufacturing because standard classifications by themselves do not give a complete picture. The underlying assumption of this paper is that investment-specific productivity growth has led to a decrease in low-skilled manufacturing employment.

Figure 2.3 plots employment by low-skilled/high-skilled and manufacturing/non-manufacturing.<sup>5</sup> It shows that low-skilled non-manufacturing employment actually

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<sup>5</sup> Data source: Bureau of Labor Statistics Current Population Survey and author's calculations.

Figure 2.3: **Employment to population (16 and over) by skill and manufacturing status**

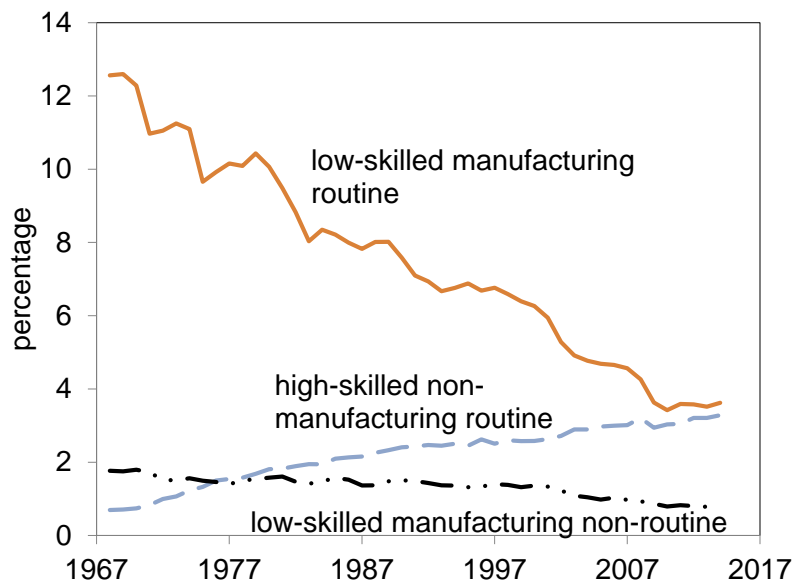


increased from 35 percent to almost 40 percent until the Great Recession. Therefore, separation of employment into low-skilled and high-skilled is not fine enough. An analogous argument applies for a classification based on manufacturing and non-manufacturing. High-skilled manufacturing employment has been stable at two to three percent. Therefore, a separation based on manufacturing and non-manufacturing is not fine enough. What has decreased is low-skilled manufacturing employment. Using the correct classification is important because the estimates of the production function depend on the classification, and results depend on the estimates.

If we add routine/non-routine, then we see in Figure 2.4 that the empirical result does not change.<sup>6</sup> The routine and non-routine classification by itself is not fine enough because high-skilled non-manufacturing routine employment has increased. Therefore, a separation based on routine and non-routine is not fine enough. We also

<sup>6</sup> With 3 classifications, there are 8 groups. I show the 3 that are enough to argue that routine is not a fine enough classification.

Figure 2.4: **Employment to population (16 and over) by skill, manufacturing status, & routineness**



see that both low-skilled manufacturing routine employment and low-skilled manufacturing non-routine employment have decreased. Therefore, low-skilled manufacturing employment is a fine enough classification.

### 2.3.2 Investment-specific productivity growth

Following Greenwood, Hercowitz, and Krusell (1997), investment-specific productivity growth is calculated as the inverse of the relative price of equipment to GDP, excluding equipment.<sup>7</sup> In the data, the relative price of equipment is measured as

$$q = \frac{\text{equipment investment deflator}}{\text{GDP deflator excluding equipment investment}}.$$

GDP deflator, excluding equipment investment, is given by

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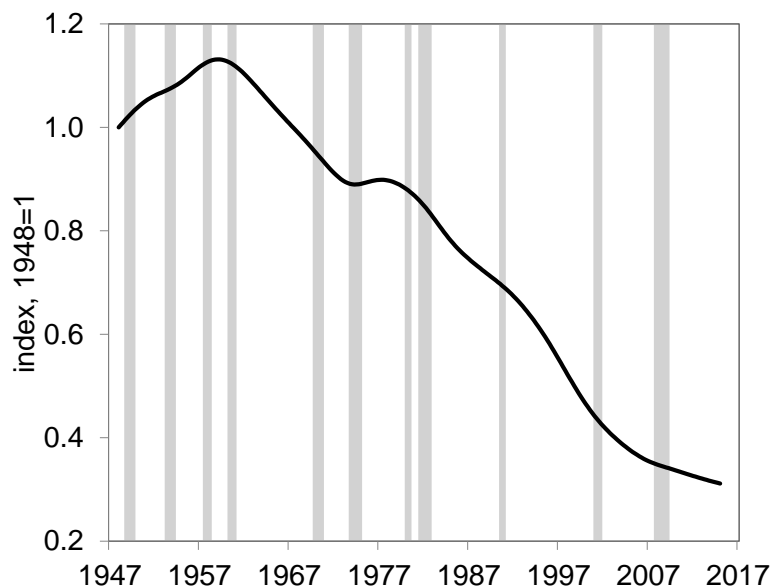
<sup>7</sup> Number of units of output for one unit of equipment.

$$\frac{\text{nominal GDP} - \text{nominal equipment investment}}{\text{real GDP} - \text{real equipment investment}}.$$

Real GDP is nominal GDP divided by the GDP deflator, and real equipment investment is nominal equipment investment divided by the equipment investment deflator.

<sup>8</sup> Note that the GDP Deflator has been adjusted so that changes in the equipment price are excluded from the GDP deflator. This adjustment is similar to that in Conesa, Kehoe, and Ruhl (2007). I plot the HP filter of relative price of equipment in Figure 2.5, with a curvature of 1,600. It is stable until mid-1960s, decreases until the early-2000s and has been stable since then. However, since it does not decrease at a constant rate, the rate of decrease in low-skilled manufacturing employment will not be constant. In the model, the base year is 1948. As we do not observe base year relative prices in the data, the series for  $q$  has been normalized such that  $q_{1948} = 1$ .

Figure 2.5: **Relative price of equipment to GDP, excluding equipment ( $q$ )**




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<sup>8</sup>  $Real\ GDP = \frac{Nominal\ GDP}{GDP\ Deflator/100}$ ,  $Real\ Equipment\ Investment = \frac{Nominal\ Equipment\ Investment}{Equipment\ Investment\ Deflator/100}$ .



## 2.4 Model

### 2.4.1 Firm's problem

Firms operate in a perfectly competitive market with a four-factor constant returns to scale production function. The four factors are capital structure  $k_s$ , capital equipment  $k_e$ , low-skilled manufacturing employment  $l_m$ , and non-low-skilled manufacturing employment  $l_n$ . The production function is given by

$$y(z, k_s, k_e, l_m, l_n) = k_s^\alpha [\mu(zl_m)^\eta + (1 - \mu)(\lambda k_e^\rho + (1 - \lambda)(zl_n)^\rho)^\frac{\eta}{\rho}]^\frac{1-\alpha}{\eta} \quad (2.1)$$

This specification is similar to that in Krusell, Ohanian, Rios-Rull, and Violante (2000), except they use high-skilled and low-skilled. I also include  $z$ , which denotes labor-augmenting productivity. The production function is Cobb-Douglas over capital structure and displays constant elasticity of substitution over the three remaining inputs.  $\mu$  and  $\lambda$  are parameters that determine income shares. The elasticity of substitution between capital equipment and non-low-skilled manufacturing employment is  $1/(1 - \rho)$ . Elasticity of substitution between low-skilled manufacturing employment and the aggregate of capital equipment and non-low-skilled manufacturing employment is  $1/(1 - \eta)$ .

When  $\eta > \rho$ , capital equipment is more substitutable with low-skilled manufacturing employment than with non-low-skilled manufacturing employment. Note that when  $\eta, \rho$  approach zero, the production function collapses to Cobb-Douglas; when  $\eta, \rho$  approach negative infinity, the production function collapses to perfect complements, and when  $\eta, \rho$  approach one, the production function collapses to perfect substitutes. Finally, given the production function and data on inputs and output, we can back out  $z$ .

Let  $s = \{z, k_s, k_e, q\}$  denote the aggregate state of the economy. The conditions that firms earn zero profits and minimize cost give rise to factor prices.

$$r_s(s) = y_{k_s}(z, k_s, k_e, l_m, l_n) \quad (2.2)$$

$$r_e(s) = y_{k_e}(z, k_s, k_e, l_m, l_n) \quad (2.3)$$

$$w_r(s) = y_{l_m}(z, k_s, k_e, l_m, l_n) \quad (2.4)$$

$$w_n(s) = y_{l_n}(z, k_s, k_e, l_m, l_n) \quad (2.5)$$

### 2.4.2 Household problem

The representative household chooses consumption  $c$ , structure investment  $x_s$ , equipment investment  $x_e$ , low-skilled manufacturing employment  $l_m$ , and non-low-skilled manufacturing employment  $l_n$ , to maximize the expected discounted value of lifetime utility. The final good can be used for consumption, structure investment or equipment investment.

Given a sequence for  $z$ , a sequence for the relative price of equipment  $q$ , interest rate functions  $r_s(s), r_e(s)$ , and wage rate functions  $w_r(s), w_n(s)$ , the household solves the following problem:

$$v(s) = \max_{c(s), k'_s(s), k'_e(s), l_m(s), l_n(s)} \log c(s) + \psi \log(1 - l_m(s) - l_n(s)) + \beta v(s') \quad (2.6)$$

subject to

$$c(s) + x_s(s) + x_e(s) = r_s(s)k_s + r_e(s)k_e + w_r(s)l_m(s) + w_n(s)l_n(s) \quad (2.7)$$

$$x_s(s) = k'_s(s) - (1 - \delta_s)k_s \quad (2.8)$$

$$x_e(s) = q[k'_e(s) - (1 - \delta_e)k_e] \quad (2.9)$$

$$c(s), k'_s(s), k'_e(s), l_m(s), l_n(s) \geq 0 \quad (2.10)$$

$$l_m(s) + l_n(s) \in [0, 1]. \quad (2.11)$$

I assume a standard time-separable utility function over consumption and leisure.  $\psi$  determines the preference for leisure.  $\beta$  is the time discount factor where  $0 < \beta < 1$ .  $r_s(s)$  and  $r_e(s)$  are the interest rate functions for capital structure and capital equipment.  $w_r(s)$  and  $w_n(s)$  are the wage rate functions for low-skilled manufacturing employment and non-low-skilled manufacturing employment.

The consumption good is the numeraire. Consumption and capital structure can be produced from the final output on a one-to-one basis. Capital structure evolves according to equation (2.8). However, that is not the case for capital equipment. One unit of capital equipment can be produced from  $q$  units of final output. This specification is similar to that of Greenwood, Hercowitz, and Krusell (1997), Krusell, Ohanian, Rios-Rull, and Violante (2000), and Conesa, Kehoe, and Ruhl (2007). Capital equipment

evolves according to equation (2.9). The underlying assumption is that the relative price of equipment affects only capital equipment investment. This is also referred to as investment-specific productivity growth. The equivalence between the relative price of equipment and investment-specific growth is proved in the appendix.

### 2.4.3 Recursive competitive equilibrium

Given capital structure  $k_s$ , capital equipment  $k_e$ , a sequence for productivity  $\{z\}$ , and a sequence for the relative price of equipment  $\{q\}$ , a recursive competitive equilibrium is interest rate functions  $r_s(s), r_e(s)$ , wage rate functions  $w_r(s), w_n(s)$ , value function  $v(s)$ , and policy functions  $k'_s(s), k'_e(s), l_m(s), l_n(s), c(s)$ , such that

- given  $r_s(s), r_e(s), w_r(s), w_n(s)$ , policy functions  $k'_s(s), k'_e(s), l_m(s), l_n(s), c(s)$  solve the household's problem.
- from the firm's problem, we have factor prices  $r_s(s), r_e(s), w_r(s), w_n(s)$ .
- markets clear

$$y(z, k_s, k_e, l_m, l_n) = c(s) + k'_s(s) - (1 - \delta_s)k_s + q[k'_e(s) - (1 - \delta_e)k_e]. \quad (2.12)$$

Note that, in this environment, the household chooses labor. That is labor is assumed to be elastic. In equilibrium, wages in low-skilled manufacturing employment and non-low-skilled manufacturing employment will be equalized. In Krusell, Ohanian, Rios-Rull, and Violante (2000), labor is assumed to be inelastic, and wages are pinned down given exogenous labor endowment.

### 2.4.4 Social planner's problem

Instead of solving for the recursive competitive equilibrium, we can solve the following equivalent social planner's problem:

$$v(s) = \max_{c(s), k'_s(s), k'_e(s), l_m(s), l_n(s)} \log c(s) + \psi \log(1 - l_m(s) - l_n(s)) + \beta v(s')$$

subject to

$$\begin{aligned}
c(s) + x_s(s) + x_e(s) &= y(z, k_s, k_e, l_m, l_n) \\
x_s(s) &= k'_s(s) - (1 - \delta_s)k_s \\
x_e(s) &= q[k'_e(s) - (1 - \delta_e)k_e] \\
c(s), k'_s(s), k'_e(s), l_m(s), l_n(s) &\geq 0 \\
l_m(s) + l_n(s) &\in [0, 1].
\end{aligned}$$

The social planner's problem is equivalent to the recursive competitive equilibrium by setting factor prices equal to their respective marginal products.

$$\begin{aligned}
r_s(s) &= y_{k_s}(z, k_s, k_e, l_m, l_n) \\
r_e(s) &= y_{k_e}(z, k_s, k_e, l_m, l_n) \\
w_r(s) &= y_{l_m}(z, k_s, k_e, l_m, l_n) \\
w_n(s) &= y_{l_n}(z, k_s, k_e, l_m, l_n)
\end{aligned}$$

## 2.5 Quantitative analysis

### 2.5.1 Parameters specified before calibration

The parameters specified before calibration are presented in Table 2.1. A period is a quarter, and therefore, the discount rate  $\beta$  is set to 0.990. Annual average depreciation rates from 1948 to 1966 were computed for structures and equipment at 0.025 and 0.125, respectively, using data from the Bureau of Economic Analysis. The annual averages have been adjusted to their respective quarterly rates.

Table 2.1: **Parameters specified before calibration**

	Variable	Parameter
$\beta$	Discount rate	0.990
$\delta_s$	Structure depreciation	0.006
$\delta_e$	Equipment depreciation	0.033

## 2.5.2 Calibration

There are six remaining parameters:  $\eta, \rho, \psi, \mu, \lambda$ , and  $\alpha$ . The parameters and target moments are presented in Table 2.2. The two key parameters in the production function are  $\eta$  and  $\rho$ .  $\eta$  is calibrated such that low-skilled manufacturing employment in the model matches the actual level of low-skilled manufacturing employment in the first quarter of 2015, which was 3.2 percent.  $\rho$  is calibrated such that total employment in the model does not change between the initial and final steady states. Note that, in this economic environment, a change in the relative price of equipment is not neutral. That is, a change in  $q$  can lead to a change in total employment.

Table 2.2: **Calibrated parameters**

Parameter		Target		
$\eta$	Elasticity ( $l_m$ and $k_e, l_n$ )	0.83	$l_m$ in 2015	3.2
$\rho$	Elasticity ( $l_n$ and $k_e$ )	-0.09	Neutral employment	44.2
$\alpha$	Share of capital structure	0.23	Average $k_s/y$	9.1
$\psi$	Share of leisure	0.99	Average $l_m$	12.6
$\mu$	Share of low-skilled manufacturing	0.46	Average $l_n$	33.5
$\lambda$	Share of capital equipment	0.24	Average $w(l_m + l_n)/y$	64.5

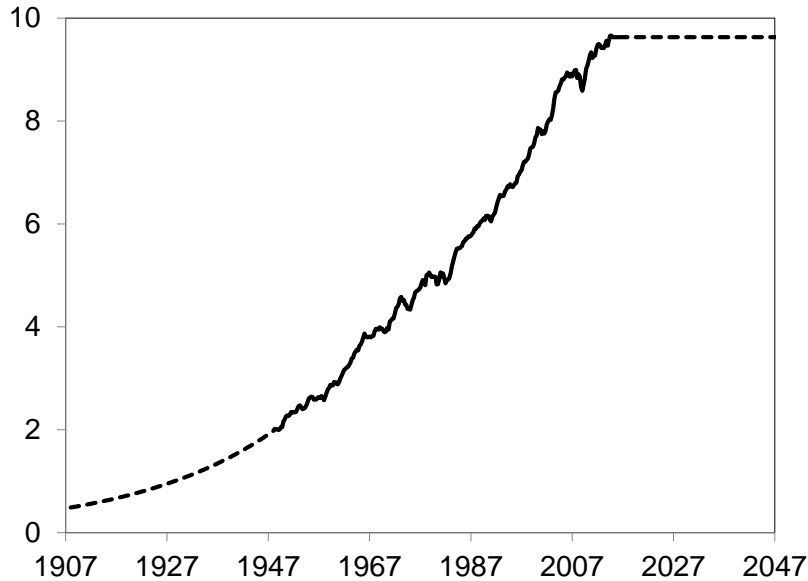
The four share parameters are calibrated to match the average of capital structure to output, the average of low-skilled manufacturing employment to population (16 and over), the average of non-low-skilled manufacturing employment to population (16 and over), and the average of labor share of output from 1948 to 1966. Finally, given parameter estimates and factor inputs and output, a sequence for  $z$  is backed out from the production function. An average productivity growth rate of  $g_z = 0.009$  was computed from the average growth rate of  $z$  from 1948 to 1966.

Table 2.3: **Elasticity estimates in the literature**

Paper	Classification	$\eta$	$\rho$
Benchmark	Manufacturing and skill	0.83	-0.09
KORV (2000)	Skill	0.40	-0.50
Eden and Gaggli (2014)	Routine	0.85	0.23

Elasticity estimates for  $\eta$  and  $\rho$  are compared to those in the literature in Table 2.3. As mentioned above, Krusell, Ohanian, Rios-Rull, and Violante (2000) use the low-skilled/high-skilled classification, while Eden and Gaggl (2015) use the routine/non-routine classification. My estimates are closer to that of Eden and Gaggl (2015) than to those of Krusell, Ohanian, Rios-Rull, and Violante (2000). This is a result of differences in estimation methods and differences in the classification.

Figure 2.6: **Aggregate Productivity ( $z$ )**



### 2.5.3 Numerical experiments

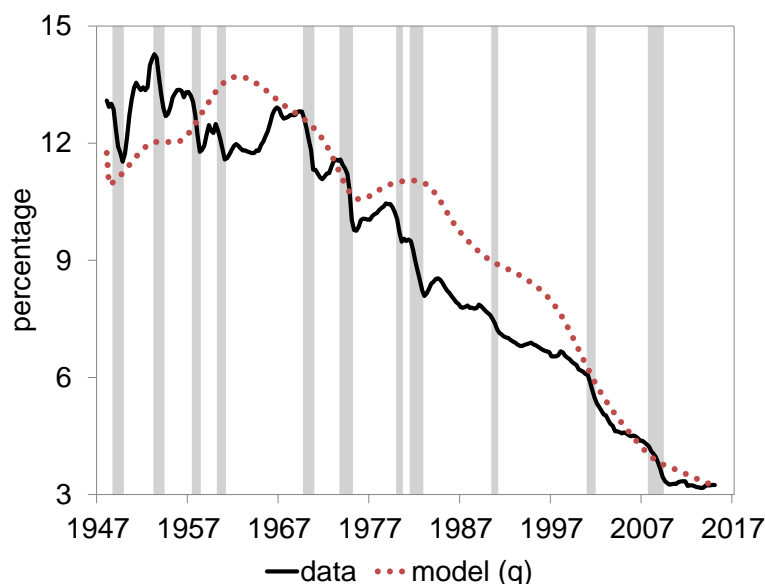
Given the model specified above, I conduct two numerical experiments. First, the transition is computed with the relative price of equipment observed in the data, (Figure 2.5), fed as deterministic process. Labor-augmenting productivity is assumed to grow at a constant rate  $g_z$  until 2015. The second case adds labor-augmenting

productivity backed out from data, (Figure 2.6).<sup>9</sup>

Note that Eden and Gaggl (2015) also compute a transition with the decrease in relative price of equipment, with a similar production function using routine employment instead of low-skilled manufacturing employment. They compute the transition to study the implications for welfare, the labor share and its composition. I compute the transition with the relative price of equipment to study the decrease in low-skilled manufacturing employment observed during recessions.

## 2.6 Results

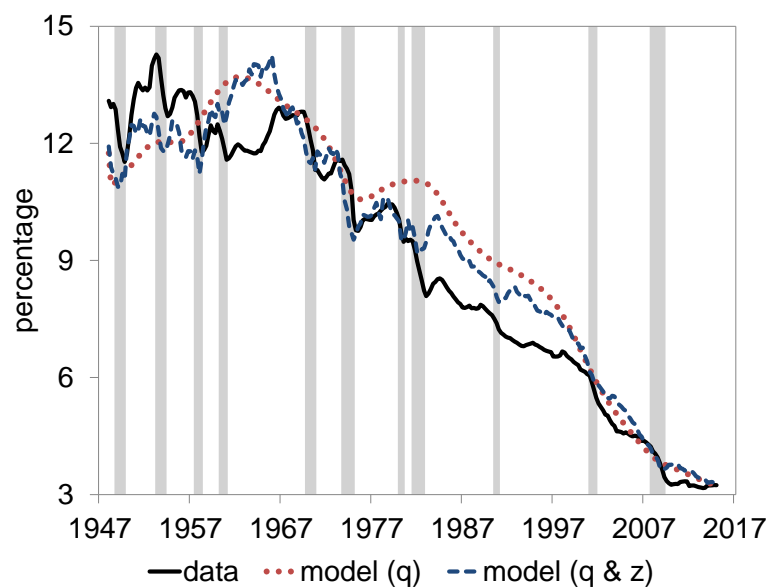
Figure 2.7: **Low-skilled manufacturing employment to population (16 and over)**



In Figure 2.7, I compare transition paths for low-skilled manufacturing employment between the data and the model for the first numerical experiment, in which I feed

<sup>9</sup> Data sources: Bureau of Economic Analysis, Bureau of Labor Statistics Current Population Survey, Bureau of Labor Statistics Current Employment Survey, and author's calculations.

Figure 2.8: **Low-skilled manufacturing employment to population (16 and over)**



in only the relative price of equipment. Labor-augmenting productivity is assumed to grow at a constant rate until 2015.

The model is calibrated to match the level of low-skilled manufacturing employment in 2015. With only the relative price of equipment, we see a smooth decrease in low-skilled manufacturing employment starting from the late 1960s. The reason for the decrease in low-skilled manufacturing employment is as follows. A decrease in the relative price of equipment leads to an increase in capital equipment. Because capital equipment is more substitutable with low-skilled manufacturing employment than with non-low-skilled manufacturing employment, we observe a decrease in low-skilled manufacturing employment. In this experiment, only 18 percent of this decrease in low-skilled manufacturing employment is observed during recessions because the total duration of recessions is shorter than the total duration of non-recessions.

Next, I feed in labor-augmenting productivity in addition to the relative price of



equipment and compute the transition. In Figure 2.8, I compare transition paths for low-skilled manufacturing employment between the data and the model. Now we observe most of the decrease in low-skilled manufacturing employment during recessions (52 percent). There are two reasons for this result. First, the rate of decrease in the relative price of equipment is not constant. It has been stable since the early-2000s. The second reason goes back to the example given by Rogerson (2014). A drop in labor-augmenting productivity leads to a recession resulting in a drop in low-skilled manufacturing employment. The post-recession recovery is offset by the decreasing trend of low-skilled manufacturing employment.

However, compared to the data, the model does have somewhat of a trend during non-recessionary periods starting from the late 1980s. Table 2.4 decomposes the decrease in low-skilled manufacturing employment into the net decrease during recessions and non-recessions.

Table 2.4: **Net decrease in low-skilled manufacturing employment**

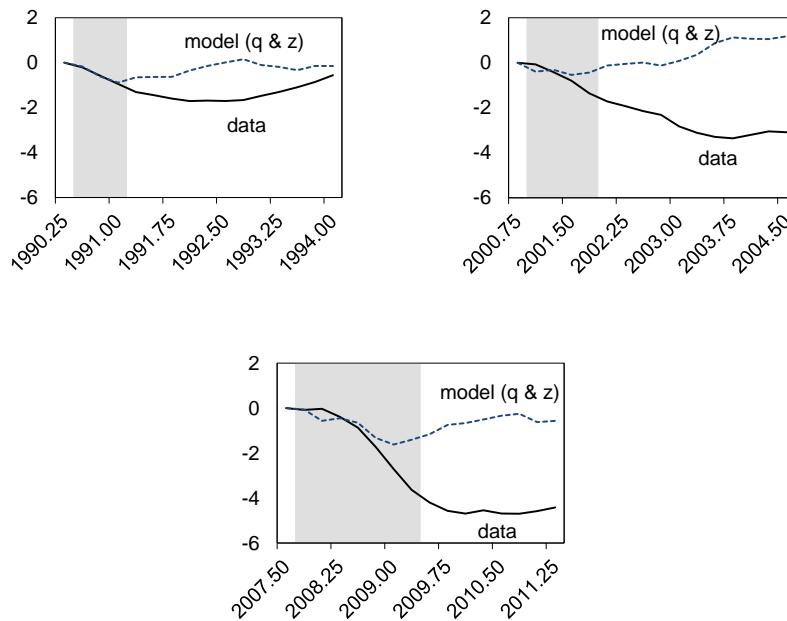
Percentage points	Data	Model (q)	Model (q & z)
Recession	-6.9	-1.8	-5.1
Non-recession	-2.8	-8.1	-4.8
Total	-9.7	-9.9	-9.9

### 2.6.1 Implications for jobless recoveries

In Figure 2.9, I compare total employment in the data and in the model in percentage-point change from its pre-recession level. Total employment in the model recovers much faster than total employment in the data. Although the model generates most of the decrease in low-skilled manufacturing employment observed during recessions, we do not observe a slow recovery of total employment.

Jaimovich and Siu (2014) and Morin (2014) study the decrease in routine employment (instead of low-skilled manufacturing employment) and find decreases concentrated in recessions with productivity shocks. However, Jaimovich and Siu (2014) also generate jobless recoveries as a result of search frictions. Morin (2014) generates jobless recoveries as a result of hiring costs. I find that most of the decrease in low-skilled

Figure 2.9: **Employment to population, 16 and over (percentage points from pre-recession level)**



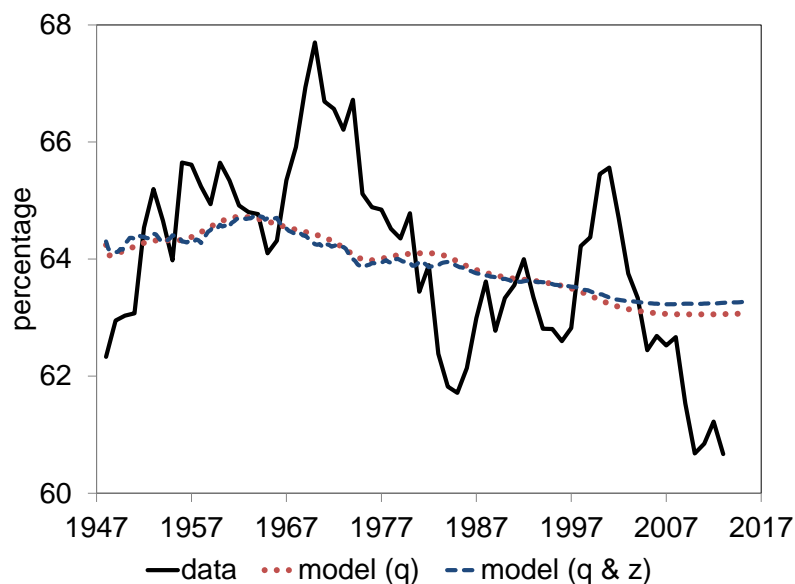
manufacturing employment is observed during recessions is without jobless recoveries. Note that we observe jobless recoveries in recessions post-1990. However, we observe the decrease in low-skilled manufacturing employment starting from the 1960s.

### 2.6.2 Implications for labor share

The computed transition path also has implications for the labor share, which is computed as total labor income divided by output. The labor share in the data increased between 1948 and the 1960s and has decreased since then. The transition path in the model is qualitatively consistent with the data, as shown in Figure 2.10.<sup>10</sup>

As a result of strong substitutability between equipment and non-low-skilled manufacturing employment ( $\eta = 0.83$ ) and weak complementarity between capital equipment and non-low-skilled manufacturing employment ( $\rho = -0.09$ ), a fall in the relative price of equipment puts more downward pressure on the wages low-skilled manufacturing

<sup>10</sup> Data sources: 2013 Economic Report of the President and author's calculations

Figure 2.10: **Labor share**

than upward pressure on the wages of non-low-skilled manufacturing. In equilibrium, wages are equalized. However, the overall increase in the wages is weakened as a result of strong substitutability. As output increases more than wages, we observe a decrease in labor share from the 1960s (total employment is almost constant). This finding is consistent with that of Karabarbounis and Neiman (2014), Eden and Gaggl (2015), and Morin (2014). I further contribute to this insight by showing that the relative price of equipment can also explain the increase in labor share observed between 1948 and the 1960s.

## 2.7 Conclusion

Low-skilled manufacturing employment as a percentage of population (16 and over) decreased by 9.7 percentage points from 1967 to 2015 for the U.S. economy. 71 percent

of this decrease is observed during recessions. I propose a theory whereby the interaction of investment-specific productivity and labor-augmenting productivity lead to 52 percent of the decrease observed during recessions. I assume a production function in which capital equipment substitutes low-skilled manufacturing employment more than non-low-skilled manufacturing employment. I back out changes in the relative price of equipment and labor-augmenting productivity from the data. Then, I feed the relative price of equipment and labor-augmenting productivity into a frictionless general equilibrium model and compute transition paths for my model. I compare low-skilled manufacturing employment between the model and the data. My model generates most of the decrease in low-skilled manufacturing employment observed during recessions, but there are no jobless recoveries. Furthermore, the movement in the labor share is qualitatively consistent with the data.

## Chapter 3

# Lack of Firm entry and the Slow Recovery of the U.S. Economy after the Great Recession

### 3.1 Introduction

This paper studies firm entry in the United States during the Great Recession and its subsequent years. We show that besides the slow recovery of employment, the recovery after the Great Recession is also characterized by the slow recovery of firm entry. Using data from the Business Dynamics Statistics (U.S. Census), we show that both the Double-Dip Recession in the 1980s and the Great Recession featured a substantial drop in the number of firms (see Figures 3.1 and 3.2).<sup>1</sup> However, when comparing the recovery in the number of firms, we verify that it has been remarkably slower after the Great Recession.

We then link the slow recovery of firm entry to the slow recovery of employment, which has been the focus of many researchers in recent years.<sup>2</sup> We consider a counterfactual exercise where we quantify the effect of lack of firm entry on employment,

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<sup>1</sup> There were two recessions in the early 1980s, but we are treating them as one. We compare the Great Recession to the Double-Dip Recession because both feature a similar drop in the magnitude of output and employment.

<sup>2</sup> E.g. Elsby et al. (2011), Jaimovich and Siu (2014), and Haltiwanger, Jarmin and Miranda (2013).

and show that it accounts for 22 percent of the difference between the actual level of employment per labor force participant in March 2012 and its pre-recession level, in March 2007. This result is consistent with recent findings in the labor economics literature, which shows that low probabilities of finding jobs play an important role in explaining fluctuations in unemployment in the business cycle frequency (e.g., Shimer (2008)).

Motivated by the empirical facts mentioned above, we then extend the industry equilibrium framework of Hopenhayn (1992) by adding aggregate uncertainty in productivity and assess how firm entry reacts to a negative supply shock and a negative demand shock. Our results show that a negative aggregate productivity shock does not generate a drop in firm entry, while a negative demand shock does. The latter causes a significant drop in firm entry, similar to the one observed during the Great Recession. However, the demand shock alone does not generate a slow recovery.

Finally, we empirically assess alternative explanations for the slow recovery. These explanations include financial constraints, offshoring, increased uncertainty at the firm level and transfers to self-employment. The empirical evidence that we provide contradicts such explanations.

**Related Literature:** our work is related to studies that have focused on firm dynamics and the Great Recession. Siemer (2014) argues that in a model with firm entry and financial constraints, a large financial shock curtails young small firms more relative to large old ones. The author then argues that a financial shock can result in a long lasting recession caused by a “missing generation” of entrants. The same distinction between young and old firms is made in Mehrotra and Sergeyev (2015) and Dyrda (2014), but they do not analyze entry and exit of firms. These studies are different from ours because we do not focus on the drop of firm entry, but on the slow recovery of firm entry. As we show, a standard model that generates a drop in firm entry would predict that firm entry should increase above trend after few periods following the recession.

The paper proceeds as follows: in Section 2 we provide the empirical evidence; in Section 3 we describe the counterfactual exercise, where we quantify the effect of the lack of firm entry on the slow recovery of employment; in Section 4 we analyze firm entry over the business cycle using a frictionless model of firm dynamics facing

aggregate uncertainty; in Section 5 we empirically assess different hypotheses for the slow recovery of firm entry; Section 6 concludes.

Our work is preliminary and incomplete. The goal of this project is to provide both empirical evidence and a theoretical understanding on the sources driving the lack of firm entry after the Great Recession.

## 3.2 Empirical Evidence

The primary dataset used in our analysis is the Business Dynamics Statistics (BDS), published by the Center of Economic Studies in the U.S. Census Bureau. It is a publicly available dataset containing annual (mid-March) information on private businesses in the United States from 1977 to 2012 (see Haltiwanger, Jarmin and Miranda (2009) and Jarmin and Miranda (2002) for a complete description of the data). It is based on administrative records and covers most of the private non-agricultural sector of the economy. The main exclusions are self-employed individuals, employees of private households, agricultural production employees, and most government employees. BDS includes only employer firms, i.e., for a firm to be included in the BDS it must have at least one employee in its payroll.

Information is available both at the firm and establishment levels. An establishment is defined as a single physical location where production takes place, while a firm corresponds to a group of establishments linked to each other by ownership status, i.e., they operate under the control of the same firm. We consider the firm as the main economic unit, since it is the one who makes the relevant decisions about the economic activities of its own establishments.<sup>3</sup> Finally, labor force series is based on the Current Population Survey (CPS) from the Bureau of Labor Statistics (BLS).

### Drop in the number of firms and its slow recovery after the Great Recession

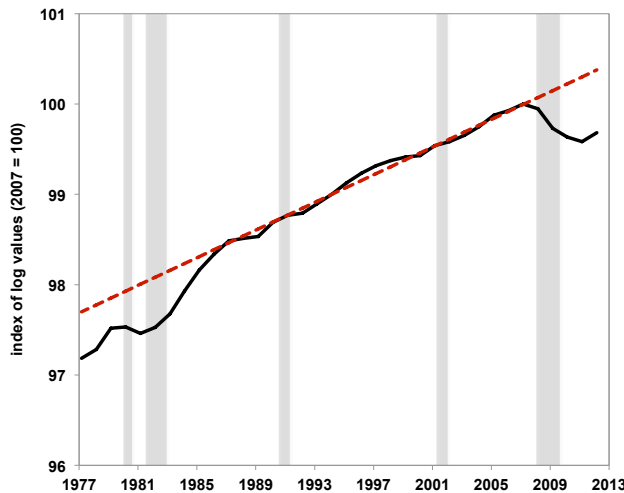
In Figure 3.2, we plot number of firms per labor force participant from 1977 until 2012. As pointed out in Luttmer (2010), the number of firms and the number of labor force

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<sup>3</sup> Our results wouldn't change if we carried out the same analysis using establishment as the main economic unit instead, because in our data, cyclical variations in the number of establishments is driven mainly by cyclical variations in the number of firms.

participants share a common trend during the period we are covering. We can see in Figure 3.2 that once we normalize the number of firms by the number of labor force participants we have a stationary series.<sup>4</sup>

Figure 3.1: Number of firms



It can be seen in Figure 3.2 that the number of firms per labor force participant dropped significantly during both the Double-Dip Recession in the early 1980s and during the Great Recession (2007-2009). From March 1979 to March 1981, the number of firms per labor force participant dropped 4.5 percent and from March 2007 to March 2010, it dropped 6 percent. The latter is the largest variation observed in the period for which we have data.

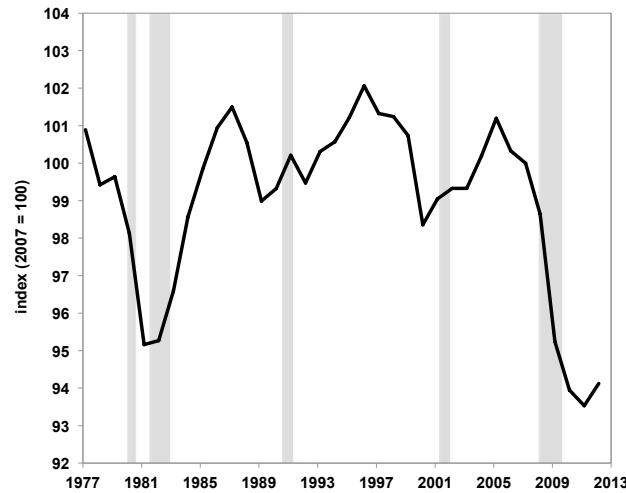
Although the Double-Dip Recession and the Great Recession have in common the fact that they both featured a significant drop in the number of firms per labor force participant (more than 4 percent each), the recovery periods following them are remarkably different. The recovery of the number of firms per labor force participant after the Great Recession has been much slower than the one observed in the early 1980's. This fact is illustrated in Figure 3.3.

According to the recession dates of the National Bureau of Economic Research (NBER), the Double-Dip Recession started in January 1980 and ended in November

<sup>4</sup> Luttmer (2010) shows that the trend is similar for the last 80 years.



Figure 3.2: Number of firms per labor force participant



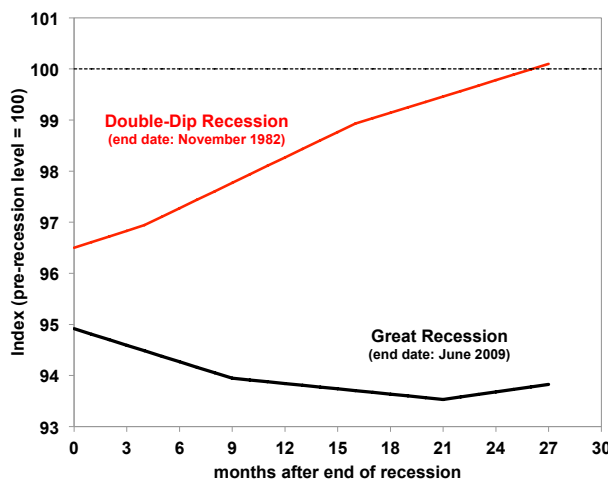
1982, and the Great Recession started in December 2007 and ended in June 2009. Since we have annual data, we take the values in March 1979 and March 2007 as the pre-recession levels.

Figure 3.3 shows that the number of firms per labor force participant increased immediately after the end of the Double-Dip Recession. After 15 months, it was only 1 percent lower than its pre-recession level, and after 25 months it had recovered completely. On the other hand, the number of firms per labor force participant continued to drop in the months after the Great Recession. Only after 20 months, it started to increase again. In March 2012, 27 months after the end of the recession, the number of firms to labor force is still 6% below trend. Therefore, even after taking into account the fact that the drop in the number of firms per labor force participant was larger during the Great Recession than during the Double Dip Recession, the recovery of the number firms per labor force participant after the Great Recession seems to be remarkably slow.

### Lack of Firm entry as the main driver

The drop in the number of firms per labor force participant observed during the Great Recession can be accounted for by either variations in the number of entering firms per labor force participant or by variations in the number of exiting firms per labor force

Figure 3.3: **Slow recovery in the number of firms per labor force participant after the Great Recession**



participant. In figure 3.4, we show how these two series evolved in the past years.

Between March 2007 and March 2009, the number of exiting firms per labor force participant increased 13 percent and the number of entering firms per labor force participant decreased 23 percent, indicating that firm entry contributed more to the initial drop in the number of firms per labor force participant during the Great Recession.

Furthermore, lack of firm entry plays a major role in accounting for the slow recovery of the number of firms per labor force participant after the Great Recession. The number of exiting firms per labor force participant returned to its pre-recession level in March 2010. However, the number of entering firms per labor force participant continued to fall, and after March 2010 it has been recovering slowly. In Figure 3.5 we plot the time series for firm entry and exit as a fraction of the size of the labor force. It shows that firm entry also dropped in the Double-Dip Recession in the early 1980s, but it recovered much faster when compared to the Great Recession.

Therefore, we can conclude that the drop in the number of entering firms per labor force participant is the main force driving both the drop in the number of firms per labor force participant during the Great Recession and its slow recovery thereafter.

Figure 3.4: Number of entering and exiting firms per labor force participant

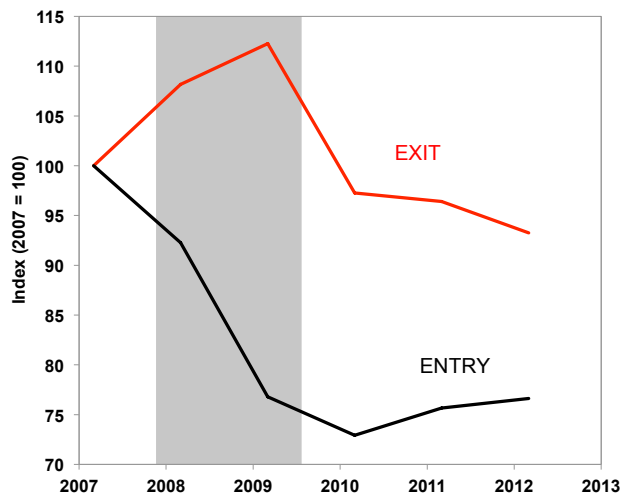
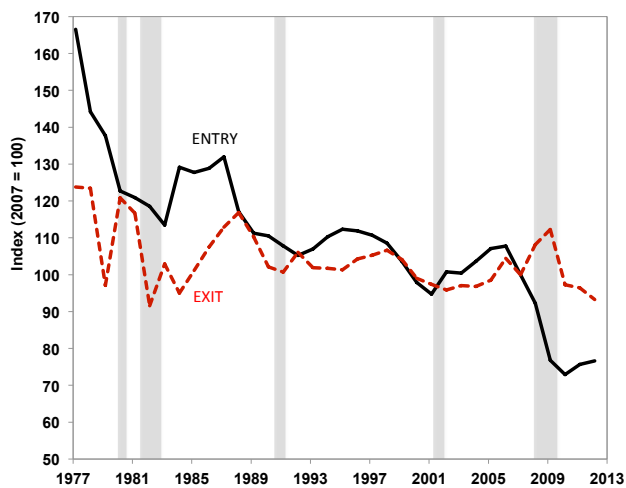


Figure 3.5: Number of entering and exiting firms per labor force participant (1977:2012)



### 3.3 Counterfactual: quantifying the impact of lack of firm entry on the slow recovery of employment

In the previous section we could see that the period following the Great Recession is also characterized by the slow recovery in the number of entering firms per labor force participant. In this section we will link the slow recovery of firm entry to the slow recovery of employment per labor force participant, a subject of much debate in the recent years (Elsby et al. (2011), Jaimovich and Siu (2014), and Haltiwanger, Jarmin and Miranda (2013)).

Despite the fact that on average younger firms have fewer employees and face lower survival rates (see Table 3.1), job creation from young firms, specially startups (age 0), is very important for the net job creation in the economy (Decker et al. (2014)). Conditional on survival, young firms show on average much higher growth rates than the more established firms (Haltiwanger et al. (2012) and Decker et al. (2014)).

Table 3.1: **Survival and growth rates of young firms**

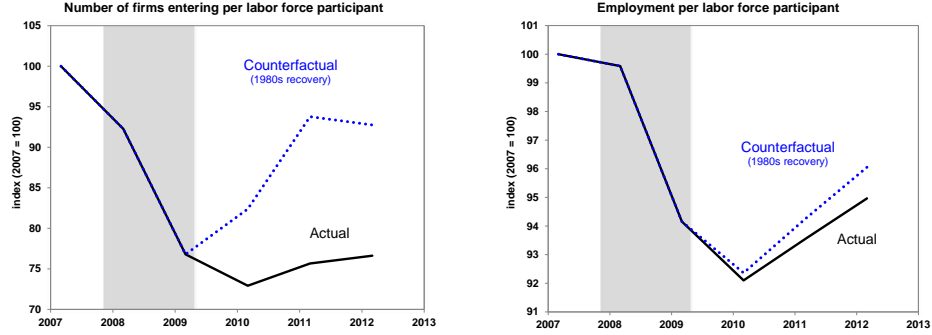
firm age (years)	average employment per firm			exit rate (%)	
	2010	2011	2012	2010-2011	2011-2012
0	6.3	5.8	5.8	25.7	23.0
1		7.5	7.3		11.8
2			8.5		

Therefore, in this section we quantify the impact of the slow recovery of firm entry on the slow recovery of employment by doing a counterfactual, where we calculate how the U.S. economy would have recovered after the Great Recession if the number of entering firms per labor force participant had recovered as it did after the Double-Dip Recession. As discussed above, since the Double-Dip Recession is the most comparable to the Great Recession in terms of magnitude, we use the recovery of the number of entering firms per labor force participant after the Double Dip Recession to discipline our analysis.

Figures 3.6a and 3.6b illustrate our exercise. Let  $entry_t^{CF}$  denote the number of

entering firms in period  $t$ , where we use subscripts  $CF$  and  $AC$  to denote the counterfactual series and the actual series, respectively, and let  $LF_t$  denote the number of labor force participants in  $t$ . We define:

Figure 3.6: Counterfactual



$$entry_t^{CF}/LF_t = entry_t^{AC}/LF_t, \quad t = 2007, 2008, 2009$$

$$\frac{entry_{2009+i}^{CF}/LF_{2009+i}}{entry_{2007}^{AC}/LF_{2007}} = \frac{entry_{1982+i}^{AC}/LF_{1982+i}}{entry_{1979}^{AC}/LF_{1979}}, \quad i = 1, 2, 3$$

For the years from 2007 to 2009, we choose the counterfactual series  $entry_t^{CF}/LF_t$  to be equal to the series that we actually observe in the data. Since our focus is on the recovery of firm entry, we decided to take the drop as given. For the periods after 2009, we assume that the number of entering firms per labor force participant recovers as it did in the early 1980s, using the pre-recession values in 1979 and 2007 as reference.

Next, we calculate the counterfactual series of employment that would result from this new series of firm entry. In order to do that, we need to take into account the differences in growth rates and survival rates between firms of different age profiles in different periods. We use the inputs in Table 3.1. For example, the average number of employees of an entrant firm (age 0) was 6.3 in 2010. Between 2010 and 2011, 25.7% of these entrant firms exited, so the survival rate of firms of age 0 in 2010 was 74.3%.

Let  $l(x, t)$  denote the average number of employees of a firm that is  $x$  years old in period  $t$ . In our example we used  $l(0, 2010) = 6.3$ . Let  $s(x, t)$  be the cumulative survival

rate that a firm of age  $x$  in  $t$  faced between  $t - 1$  and  $t$ . In our example, we used  $s(1, 2011) = 0.743$ . We define  $s(0, t) = 1$ . The counterfactual series of employment per labor force participant is given by,

$$emp_t^{CF}/LF_t = emp_t^{AC}/LF_t, \quad t = 2007, 2008, 2009$$

$$emp_t^{CF}/LF_t = emp_t^{AC}/LF_t + \sum_{i=0}^{t-2010} (entry_{t-i}^{CF} - entry_{t-i}^{AC}) \frac{l(i, t)}{LF_t} \prod_{j=0}^i s(i-j, t-j)$$

for  $t = 2010, \dots, 2012$ .

Note that we are assuming that the firms that did not enter would behave exactly as the ones that did enter. Figure 3.6b shows the counterfactual series for employment. In 2010, the lack of firm entry could explain only 3 percent of the difference between the actual value of the employment per labor force participant and its pre-recession level. However, by 2013, it could explain 22 percent of the difference. The reason for the divergence between the two series is exactly the cumulative effect of firm entry. For example, in 2012, besides taking into account the employment level of entering firms, we also need to account for the employment levels of the previous cohorts (2010 and 2011), adjusted by the survival and growth rates according to their respective age profiles.

The facts presented above lead us to analyze how a simple model of firm dynamics is capable of explaining the behavior of firm entry in the business cycle.

### 3.4 Model

We extend the industry equilibrium framework of Hopenhayn (1992) by adding aggregate uncertainty in productivity and aggregate uncertainty in the marginal rate of substitution between consumption and labor. We add aggregate uncertainty in productivity because we want to first study what happens to firm entry when there is a negative supply shock, which is represented by the aggregate productivity shock. We add aggregate uncertainty in the marginal rate of substitution between consumption and labor because we want to study what happens to firm entry when there is a negative demand shock, which is represented by the negative preference shock affecting the

marginal rate of substitution between consumption and labor.

### Firm's Problem

Upon entry, firms draw idiosyncratic productivity  $s$  from a distribution  $G(s)$  after paying sunk entry cost  $c_e$ , in units of labor. After that, idiosyncratic productivity shocks  $s$  follow a log  $AR(1)$  process:

$$\log s_{t+1} = \rho_s \log s_t + \epsilon_{t+1}^s, \quad \epsilon \sim N(0, \sigma_{\epsilon^s}^2)$$

A firm is then characterized by its idiosyncratic productivity  $s$ . Let  $\Omega$  be the distribution of firms. The aggregate state of the economy is given by aggregate productivity  $Z^A$ , aggregate preference shock  $Z^D$ , and the distribution of firms  $\Omega$ , over  $s$ . Let  $S = (Z^S, Z^D, \Omega)$  denote the aggregate state of the economy.

A firm maximizes the expected discounted value of profits, which are then passed on to households who own the firms. In our setting, this is equivalent to the firm facing a sequence of static problems. Given a decreasing returns to scale technology, the firm chooses labor in order to maximize current profits. The current profits are given by,

$$\pi(s, S) = \max_{l_f(s, S)} s Z^A l_f(s, S)^\theta - w(S) l_f(s, S)$$

where  $0 < \theta < 1$ .

Firms die exogenously with probability  $\eta$ , where  $0 < \eta < 1$ . The value of a firm with idiosyncratic productivity  $s$  is given by

$$V^f(s, S) = \pi(s, S) + \beta(1 - \eta) E_{S'} m(S') V(s', S')$$

where  $m(S') = \frac{U_c(S')}{U_c(S)}$  is the stochastic discounting factor of the representative household.

### Household Problem

A representative household faces a sequence of static problems where it chooses consumption and leisure, given  $Z^D$ ,  $w(S)$ , and  $\Pi(S)$ .

$$\begin{aligned}
& \max_{C(S), L(S)} U(C(S), 1 - L(S); Z^D) \\
& \quad s.t. \\
& C(S) = w(S)L(S) + \Pi(S) \\
& C(S) \geq 0; L(S) \in [0, 1]
\end{aligned}$$

### Recursive Competitive Equilibrium

Given initial aggregate state  $(Z_0^A, Z_0^D, \Omega_0)$ , an *equilibrium* is wage function  $w(S)$ , mass of entrants function  $\mu(S)$ , value functions for the firm  $V^f(s, S)$ , policy functions for the household  $C(S)$ ,  $L(S)$  and for the firms  $l_f(s, S)$  such that

- given  $w(S)$ , the policy functions  $C(S)$ ,  $L(S)$  solve the household problem;
- given  $w(S)$ ,  $V^f(s, S)$ , the policy function  $l_f(s, S)$  solves the firm's problem;
- the zero-profit condition holds

$$\int V^f(s, S)G(s)ds = w(S)c_e;$$

- markets clear,

$$\begin{aligned}
C(S) &= \int sZ^A l_f(s, S)^\theta (\Omega(s) + \mu(S))ds \\
L(S) &= \int l_f(s, S)(\Omega(s) + \mu(S)G(s))ds + \mu(S)c_e
\end{aligned}$$

- the distribution of firms  $\Omega$  evolves according to

$$\Omega'(B) = (1 - \eta) \int_{1\{s' \in B\}} \int f(s, s')(\Omega(s) + \mu(S)G(s))dsds'.$$

for all  $B \subset S$ .

**Remarks:** We assume that firms die exogenously with probability  $\eta$  mainly because we are focusing on lack of firm entry. Exit in this model can be endogenized by adding a fixed operating cost as in Hopenhayn (1992). While the assumption of exogenous exit doesn't drive our results, it reduces the computational burden of solving for equilibrium (see Appendix).



## Quantitative Analysis

For the functional form of the utility function, we choose

$$U(c, 1 - l; Z^D) = Z^D \log c + \psi \log(1 - l).$$

Note that in this case  $Z^D$  works as a labor wedge (Chari et al. (2008)). Aggregate preference and productivity shocks follow  $\log AR(1)$  processes,

$$\begin{aligned} \log Z_{t+1}^A &= \rho_A \log Z_t^A + \epsilon_{t+1}^A, \\ \log Z_{t+1}^D &= \rho_D \log Z_t^D + \epsilon_{t+1}^D, \end{aligned}$$

where  $\epsilon_{t+1}^A \sim N(0, \sigma_{\epsilon^A}^2)$  and  $\epsilon_{t+1}^D \sim N(0, \sigma_{\epsilon^D}^2)$ .

The labor preference parameter  $\psi$  is chosen such that the Frisch elasticity of labor with respect to the wage rate is 2.65. This is in the range used in the macro literature (Rogerson and Wallenius, 2009). The death rate  $\eta$  is chosen to be .08, which is the average exit rate of firms in the data (1977 to 2007). We set  $c_e = 0.11$  so that entrants' share of aggregate employment is equal to 3%. The rest of the parameters are standard.

Table 3.2: **Parameter values**

labor share	$\theta$	0.64
discount factor	$\beta$	0.96
death rate	$\eta$	0.08
cost of entry	$c_e$	1.36
preference for leisure	$\psi$	1.75
idiosyncratic shock persistence	$\rho_s$	0.70
idiosyncratic shock standard deviation	$\sigma_{\epsilon^s}$	0.22
persistence of aggregate stochastic processes	$\rho_A = \rho_D$	0.80
standard deviation of aggregate stochastic processes	$\sigma_{\epsilon^A} = \sigma_{\epsilon^D}$	0.01

Figure 3.7: Impulse responses of a 4 percent drop in aggregate productivity

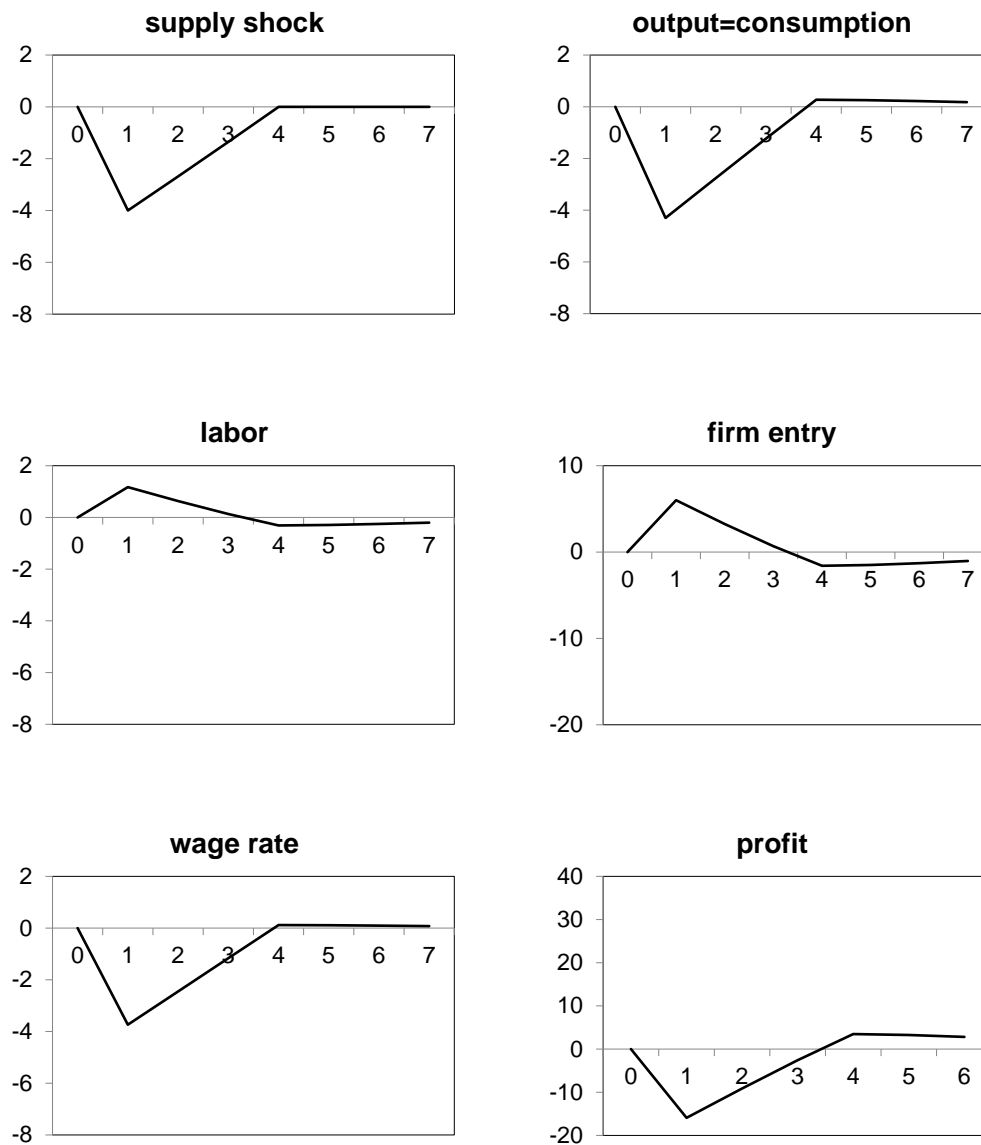
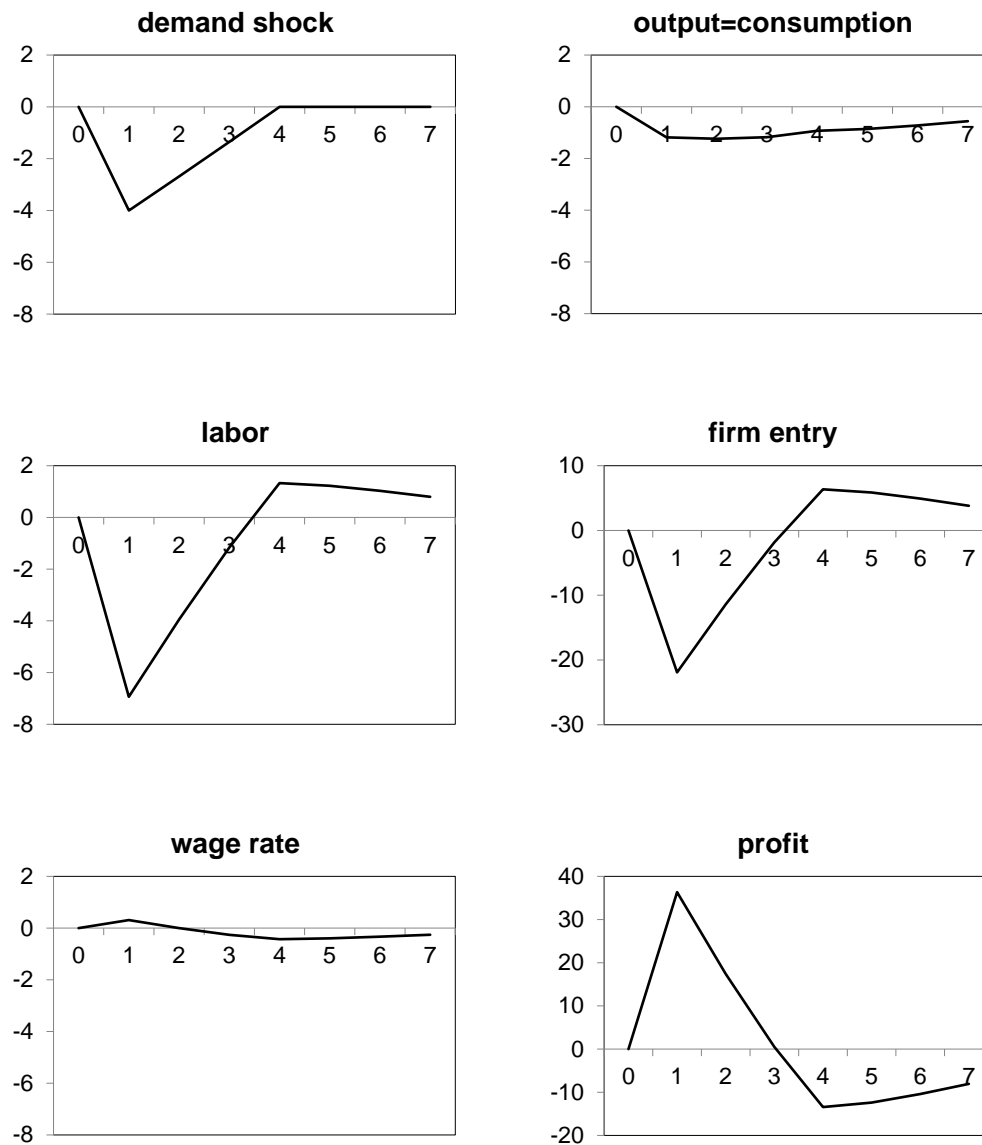


Figure 3.8: Impulse responses of a 4 percent negative demand shock



In these kinds of models where we have firm heterogeneity and aggregate uncertainty, we have to keep track of the firm distribution to solve for prices. However, that is an object with infinitely many dimensions. This leads to an algorithm similar to that used in Krusell and Smith (1998), Khan and Thomas (2003), and Clementi and Palazzo (2014). The algorithm is discussed in more detail in the Appendix.

In Figure 3.7 we show the impulse response functions resulting from a 4 percent drop in aggregate productivity. The drop in aggregate productivity leads to a similar drop in wages, which leads to an increase in firm entry. As can be seen in 3.7, output falls by approximately 4 percent and firm entry increases by almost 8 percent. We can also observe that firm entry recovers quickly after the recession. Since entry is a flow, it makes sense that it falls back to a level below trend after the recession so that the mass of firms in the economy also recovers back to trend. Therefore, we could see that in this simple model of firm dynamics, a negative productivity shock actually generates an increase in firm entry, contrary to what we observed in Great Recession.

In Figure 3.8 we show the impulse response functions resulting from a 4 percent drop in  $Z^D$ . The negative demand shock leads to a small increase in wages, which leads to a drop in firm entry. As can be seen in 3.7, output falls by approximately 1 percent and firm entry falls by almost 20 percent. We can also observe that firm entry recovers quickly after the recession. Therefore, we could see that in this simple model of firm dynamics, a negative demand shock generates a drop in firm entry similar to the one observed in the Great Recession. However, it does not generate persistence. After three periods, firm entry is already above its steady-state level.

This leads us to the next section, where we rule out possible hypotheses that have been often suggested in the literature. We do it based on empirical evidence .

## 3.5 Assessing Alternative Hypotheses

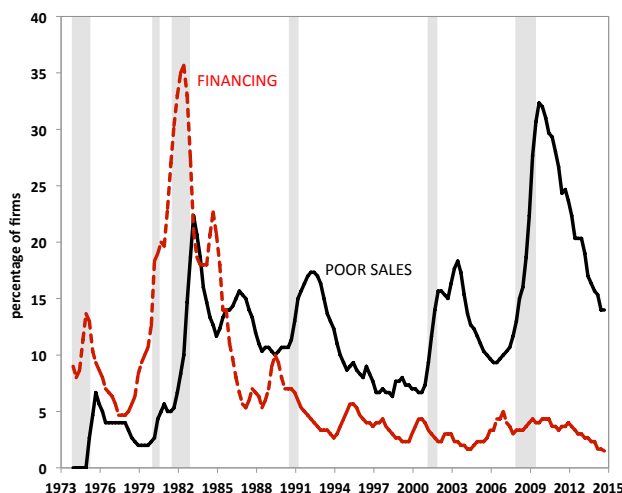
### Financial Constraints

Given the financial aspect of the Great Recession, many models that try to account for it rely on financial frictions. Following this line, Mehrotra and Sergeyev (2015) and Siemer (2014), both featuring firm dynamics, model the crisis as financial shocks. They focus on the financial needs of young firms, that borrow from commercial banks

in order to finance their initial investment (Robb and Robinson (2014)). Mehrotra and Sergeyev (2015) consider that young firms use real state as collateral in order to finance investment, so the drop in housing prices observed in the data could represent a tightening of the borrowing constraint. Siemer (2014), on the other hand, explain the slow recovery in the number of entering firms as the result of a credit crunch that followed the crisis, which reduced bank lending to new business.

In order to assess the financial constraint channel, we use the survey conducted by the National Federation of Independent Business, the Small Business Economic Trends. In this survey, small business owners are asked what is the single most important problem they are facing. The alternatives are: taxes, inflation, poor sales, financing and interest rates, cost of labor, government regulation, competition from large businesses, quality of labor, cost of insurance, and others.

Figure 3.9: **Single most important problem**



In Figure 3.9 we plot the time series for two of the alternatives: financing and interest rates, and poor sales.<sup>5</sup> Despite the fact that financing seemed to be a major issue during the Double-Dip Recession, it does not show a similar pattern in the recent crisis. We take this result as evidence that the financial constraint channel, at least in the way it has been proposed so far, is not the main driver of the slow recovery in firm entry.

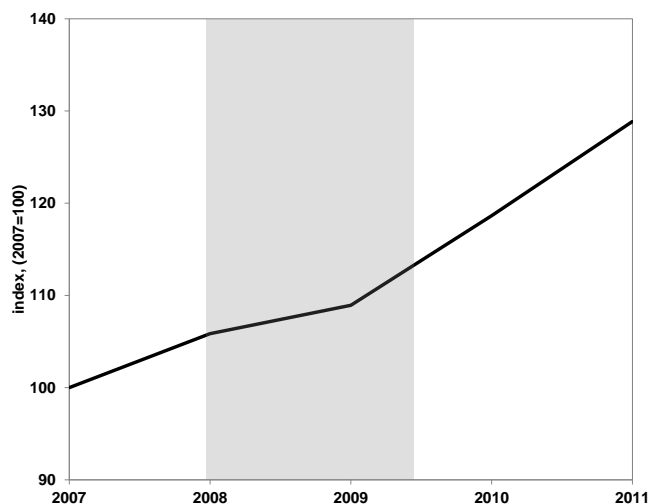
<sup>5</sup> These are the series that show higher cyclical volatility during recessions.

## Openness and Offshoring

Since the 1980s, the U.S. economy has become more open and offshoring of jobs by companies operating in the U.S. has increased. This is a factor that has been popular in explaining jobless recoveries (e.g., Waddle (2013)). We question if offshoring and increased openness is contributing to the slow recovery of firm entry. We consider two mechanisms through which offshoring and increased openness might contribute to the slow recovery of firm entry.

First, we consider a direct mechanism, where we would observe less foreign firms entering the U.S. market. However, in Figure 3.10, we plot the number of tax returns filed by foreign corporations operating in the U.S. which shows that it has continued to increase since 2007.

Figure 3.10: Number of tax returns filed by foreign corporations in the U.S.



Second, we consider an indirect mechanism, where large firms in the U.S. substituting inputs from domestic firms by foreign inputs. In this case, the lower demand for domestic inputs might reduce the incentives of new firms to enter the market. However, we observe lack of firm entry in all sectors, including sectors which are highly nontradable (e.g., construction and retail services). This can be seen in Figure ??, where we plot the number of entering firms per labor force participants for the sectors that account for most of entering firms in the economy: service; construction; retail; finance, insurance

and real estate. They account for 46%, 7%, 22%, 10% of entering firms, respectively, and together they account for 85% of firm entry.

### **Uncertainty at the firm level**

Suppose firms face idiosyncratic time varying productivity shocks as in Hopenhayn (1992). Bloom et al. (2011) study manufacturing establishments for the U.S. economy and show that the variance of idiosyncratic shocks increases during recessions. The literature refers to it as increased uncertainty at the firm level. Arellano et al. (2012) argue that increased uncertainty along with labor adjustment costs and financial frictions can generate a significant decline in output and labor, but not in labor productivity, similar to what was observed in the Great Recession.

However, in Bloom et al. (2012), we can see that in the Double-Dip Recession in the 1980s, uncertainty increased to approximately 85 percent of the level observed during the Great Recession. Therefore, an explanation to the slow recovery in firm entry that relies on increased uncertainty at the firm level must account for the Double-Dip Recession in the 1980s, when firm entry recovered relatively quick. It is then a challenge to explain why increased uncertainty would generate a slow recovery in the recent recession as compared to the Double-Dip Recession in the 1980s, unless there was some structural change that complements the increased uncertainty.

### **Self employed**

In the data we use, BDS, self-employment is not included. So it might be the case that more people are becoming self-employed, which might explain the drop in the number of new employer firms and its slow recovery. However, Figure ?? shows that the recovery in the number of self-employed after the Great Recession has been slow, which contradicts the hypothesis.

## **3.6 Conclusion**

Besides the slow recovery of output and employment, we showed that lack of firm entry is another feature of the Great Recession and its subsequent years. We have shown that the number of firms per labor force participant dropped significantly during the Great

Recession and has been recovering slowly ever since, and that lack of firm entry is the main force driving it.

We quantified the effect of the lack of firm entry on the slow recovery of employment, where we showed that it accounts for 22 percent of the lack of employment by 2012.

We then investigate how firm entry reacts to negative supply and demand shocks in a simple firm dynamics model. The supply shock does not generate a drop in firm entry, while the demand shock does. The latter causes a significant drop in firm entry, similar to the one observed during the Great Recession. However, the demand shock alone does not generate a slow recovery.

Finally, we showed how empirical evidence contradicts common explanations for the slow recovery. These explanations include financial constraints, offshoring, increased uncertainty at the firm level and transfers to self-employment.

For future work, the goal of this project is to provide both empirical evidence and a theoretical understanding on the sources driving the lack of firm entry after the Great Recession.



## Chapter 4

# The Effects of Aging on the Recovery of the U.S. Economy after the Great Recession

### 4.1 Introduction

The U.S. economy experienced the Great Recession in 2007. Since then, employment to population has not recovered for both ages 25-64 and ages 25 plus. Employment to labor force has recovered for both ages 25-64 and ages 25 plus. Labor productivity recovered faster than GDP. This paper shows that the aging population in the U.S. economy can jointly account for these 3 facts. An implication of this result is that the U.S. economy will not recover to its trend prior to the Great Recession.

The 3 facts are presented in Figure 4.1, Figure 4.2, and Figure 4.3. Figure 4.1 shows the percentage point change in employment to population from 2007 to 2017 for both ages 25-64 and 25 plus, where the 2007 level equals zero. There is an approximately 4 percentage point decrease in employment to population for both ages 25-64 and 25 plus. For ages 25-64, employment to population is almost 2 percentage points below its pre-recession level by 2017. For ages 25 plus, employment to population is almost 3 percentage points below its pre-recession level by 2017.

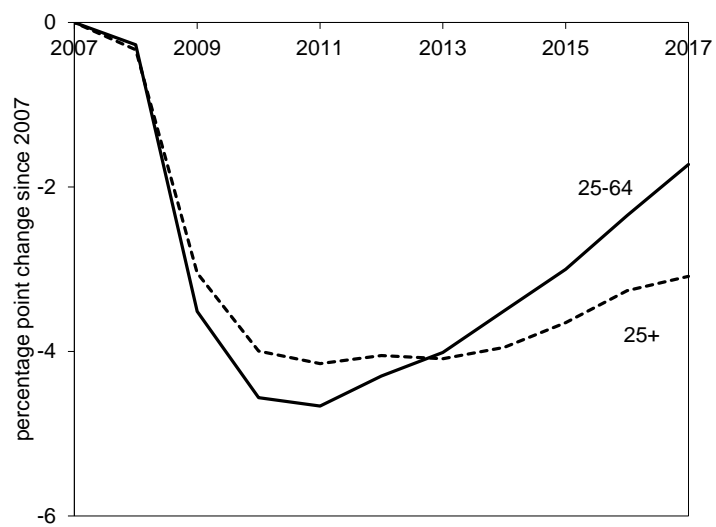
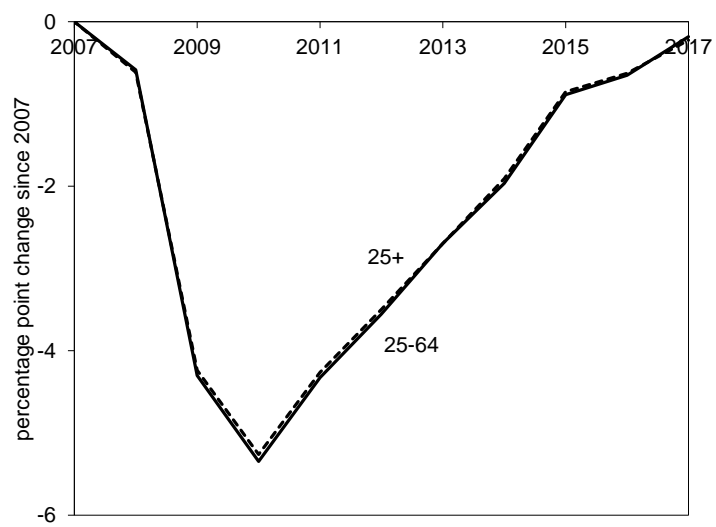
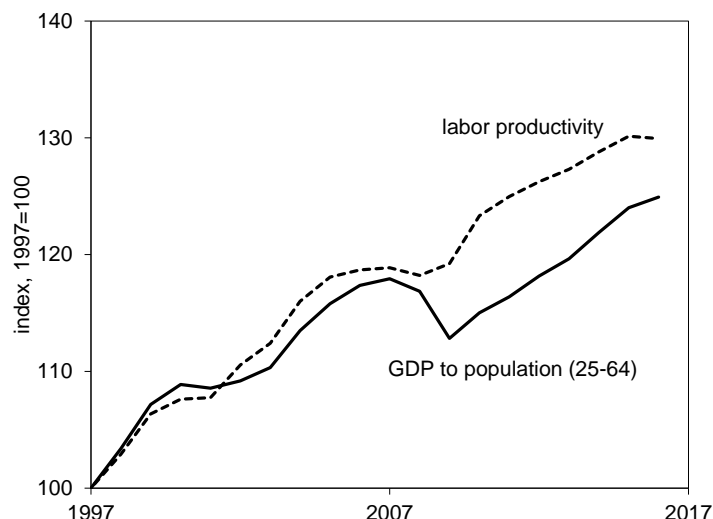
Figure 4.1: **Employment to population**Figure 4.2: **Employment to labor force**

Figure 4.2 shows the percentage point change in employment to labor force from 2007 to 2017 for both ages 25-64 and 25 plus, where the 2007 level equals zero. Employment to labor force has recovered to its pre-recession level by 2017 for both ages 25-64 and ages 25 plus.

Figure 4.3 shows GDP to population (25-64) and labor productivity (GDP to employment, 25-64) from 1997 to 2017, where the 2007 level has been normalized to 100. From 1997 to 2007, labor productivity tracks GDP to population (25-64). By 2017, they have diverged. That is, labor productivity has recovered faster than GDP to population (25-64).

Figure 4.3: **GDP and labor productivity**



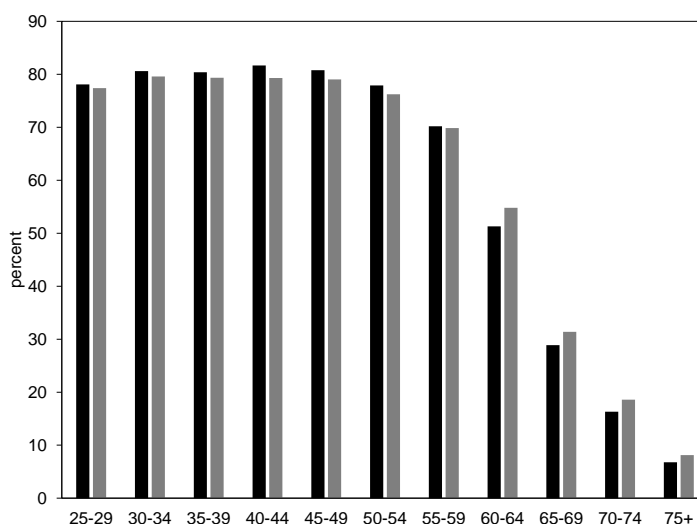
The two key mechanisms that drive the results are as follows. First, employment decreases with age for 45 years and older. Therefore, an aging population can lead to a decrease in employment to population even for ages 25-64 (and 25 plus). Furthermore, it can lead to a constant employment to labor force ratio. Second, average labor productivity increases with age until the mid 50s. Therefore, an aging population can lead to an increase in the productivity of employees. This can increase labor productivity at

a faster rate than GDP to population (25-64). To quantify the effects of these mechanisms, this paper builds an overlapping generations model where consumers choose between full employment and leisure. The labor productivity has an age specific component that is hump shaped. Disutility of employment is assumed to vary with age and is calibrated to match employment by age over the life cycle. A transition is computed with the growth rates of unborns calculated directly from data from 1909 to 2015. The results show that after 2007, there is a decrease in employment to population for both ages 25-64 and 25 plus. Furthermore, there is a divergence between labor productivity and GDP to population (25-64), consistent with the data.

## 4.2 Counterfactual: how big are the numbers?

Figure 4.4 shows that employment decreases with age after 45 years and older. Therefore, an aging population implies a lower employment to population ratio (25-64 and 25 plus) and a constant employment to labor force ratio (25-64 and 25 plus), as discussed above.

Figure 4.4: **Employment by age (2007 and 2017)**



To quantify the effect of the aging population on employment, the following counterfactual exercise studies how employment would have recovered if the population composition did not change after 2007. That is, suppose the fraction of  $\{25 - 29, 75+\}$  year olds remained constant since 2007. We observe employment to population for  $i \in \{25 - 29, \dots, 75+\}$  and  $t = 2007, \dots, 2017$ . Counterfactual employment numbers are computed as follows:

$$emp_{i,t}^{counterfactual} = \frac{emp_{i,t}^{actual}}{pop_{i,t}} \times pop_{i,2007} \quad (4.1)$$

Figure 4.5: **Counterfactual: employment to population (25-64)**

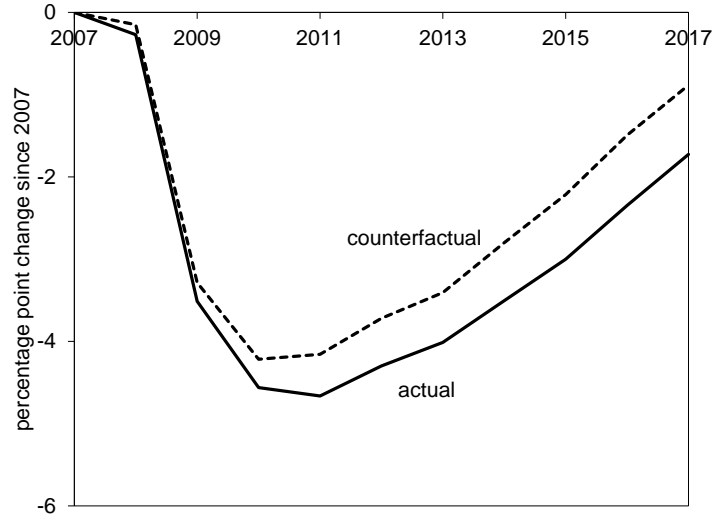
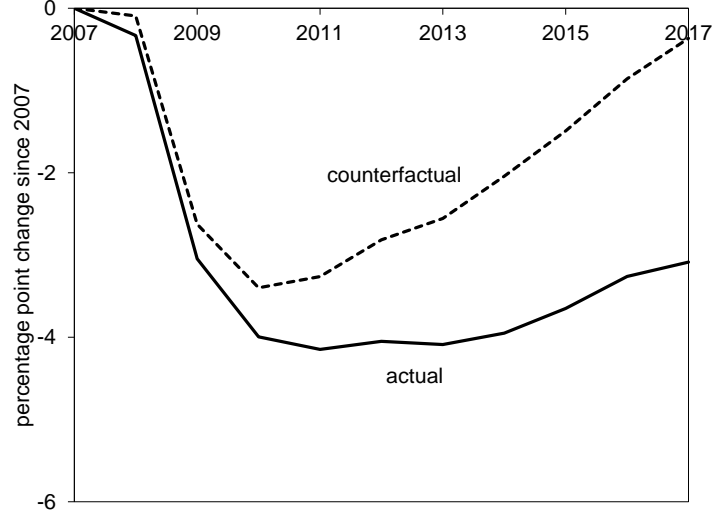


Figure 4.5 and Figure 4.6 compare counterfactual employment with actual employment for ages 25-64 and 25 plus. Figure 4.5 shows that if the composition of the population did not change since 2007, the level of employment would have been 0.84 percentage points above its actual level in 2007. Figure 4.6 shows that employment would have almost recovered to its 2007 level by 2017 if the composition of the population did not change after 2007. These two counterfactuals illustrate that aging has a first order effect on the recovery of employment (and possibly output and labor productivity) after

2007.

Figure 4.6: **Counterfactual: employment to population (25+)**



### 4.3 Model

The model is a discrete time, small open economy, overlapping generations model with ex-ante heterogeneous consumers, where consumers differ in age, assets, and labor productivity. Consumers are born at age 25; the maximum age is 100; and a period is 1 year. The growth rate of 25 year olds in period  $t$  is given by  $g_{nt}$ . As discussed in the introduction, changes in  $g_{nt}$  can explain the 3 facts. To simplify the analysis, the growth rate of 25 years olds in the model is assumed to equal the growth rate of unborns observed in the data, with a 25 year delay.

Consumers face idiosyncratic earnings risk and supply indivisible labor at a cost  $\chi_j$ . The idiosyncratic state  $(j, a, \eta)$  is given by the age  $j$ , the level of assets  $a$ , and the idiosyncratic productivity component  $\eta$ . Consumers cannot borrow. That is,  $a' \geq 0$ . The idiosyncratic productivity  $\eta$  is persistent and follows a log  $AR(1)$  process. Age  $j$  is associated with disutility of working  $\chi_j$ , deterministic productivity  $\epsilon_j$ , and conditional

survival probability  $\psi_j$ . When consumers die, their assets and asset earnings are distributed uniformly as accidental bequests, given by  $Tr$ .

#### 4.3.1 Consumer problem

The consumer problem is given by the following value function:

$$V_t(j, a, \eta) = \max_{c, a', d \in \{0, 1\}} \frac{[c^\gamma (\bar{l} - d(1 + \chi_j))^{1-\gamma}]^{1-\sigma}}{1 - \sigma} + \psi_j \beta E_{\eta'|\eta} V_{t+1}(j + 1, a', \eta')$$

*s.t.*

$$c + a' = dw_t \exp(\eta) \epsilon_j + (1 + r)a + Tr_t$$

$$c, a' \geq 0$$

#### 4.3.2 Competitive equilibrium

Given a sequence of growth rates for 25 year olds  $\{g_{nt}\}_{t=1}^\infty$ , initial condition for the distribution of agents  $\Omega_0$ , a competitive equilibrium is a sequence of individual value and policy functions for the households  $\{V_t, c_t, a'_t, d_t : \mathbb{R} \times \mathbb{R} \times I \rightarrow \mathbb{R}\}_{t=1}^\infty$ , a sequence of production plans for the firm  $\{K_t, L_t\}_{t=1}^\infty$ , a sequence of prices  $\{w_t\}_{t=1}^\infty$ , and a sequence of measures  $\{\Omega_t\}_{t=1}^\infty$  such that, for all  $t$ , the following hold:

- Given prices, consumers solve their respective problems;
- Wage rate  $w_t$  satisfies

$$w_t = (1 - \alpha)(K_t/L_t)^\alpha;$$

- Law of motion for  $\Omega_t$  is consistent with the policy functions and the exogenous productivity process; and
- Markets clear;

$$L_t = \int d(j, a, \eta) \exp(\eta) \epsilon_j \Omega_t(dj \times da \times d\eta)$$

## 4.4 Calibration

Figure 4.7: **Growth rate of unborns**

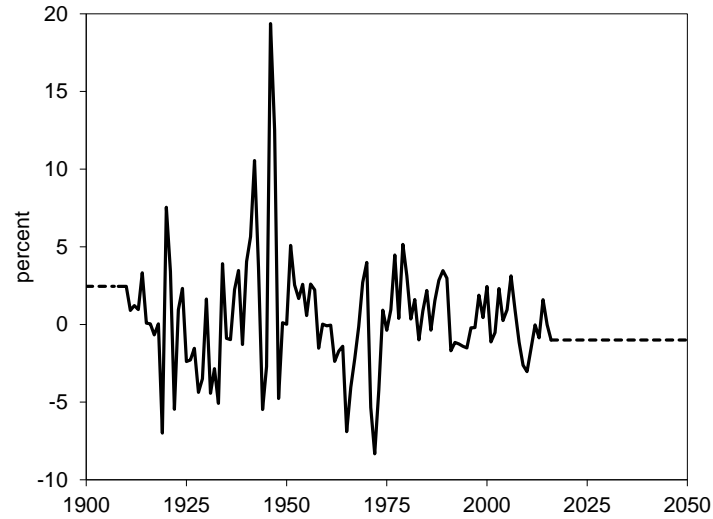
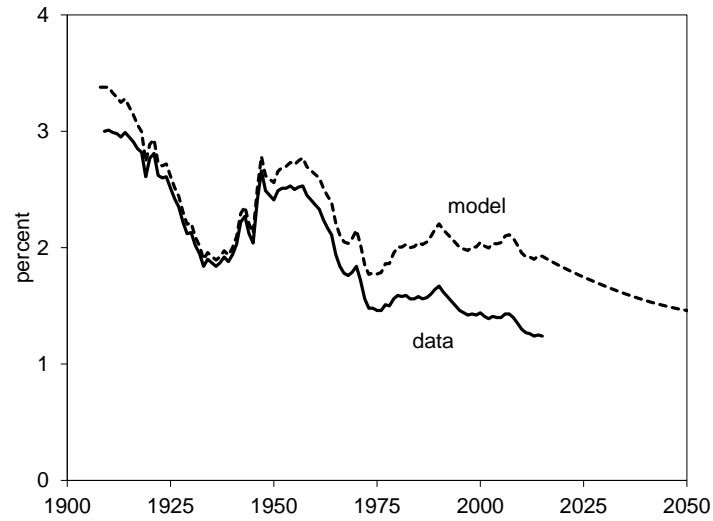


Figure 4.7 shows the growth rate of unborns computed directly from the data. As mentioned above, to simplify the analysis, this paper assumes the growth rate of 25 year olds in the model is equal to the growth rate of unborns with a 25 year delay. Figure 4.8 compares the birth rate observed in the data and the model. The birth rate in the model tracks what is observed in the data until the early 1970s. After the 1970s, the birth rate in the model is higher than what is observed in the data. This is because, in the data survival rates have increased. The model assumes a constant age-specific survival probability. Note that results are not sensitive to this assumption because the model can account for all 3 facts for both ages 25-64 and ages 25 plus.



Figure 4.8: **Birth rate: model vs data**

As discussed in the section above, employment decreases with age for 45 years and older. The disutility by age  $\chi_j$  is calibrated to match the life cycle profile of employment in 2007 (Figure 4.9). Figure 4.10 shows the age efficiency parameter  $\epsilon_j$  computed from estimates by Conesa et al. (2017) and Ludwig and Krueger (2015). This paper uses numbers estimated by Conesa et al. (2017). Results are not sensitive to either estimates.

Figure 4.9: Disutility by age

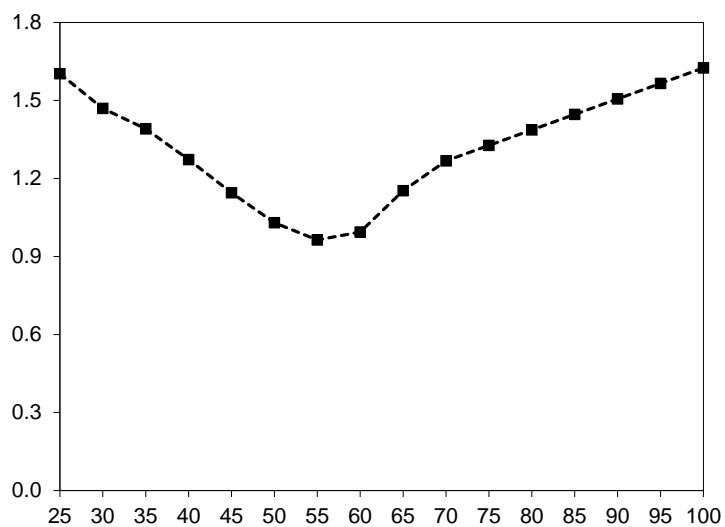


Figure 4.10: Age efficiency

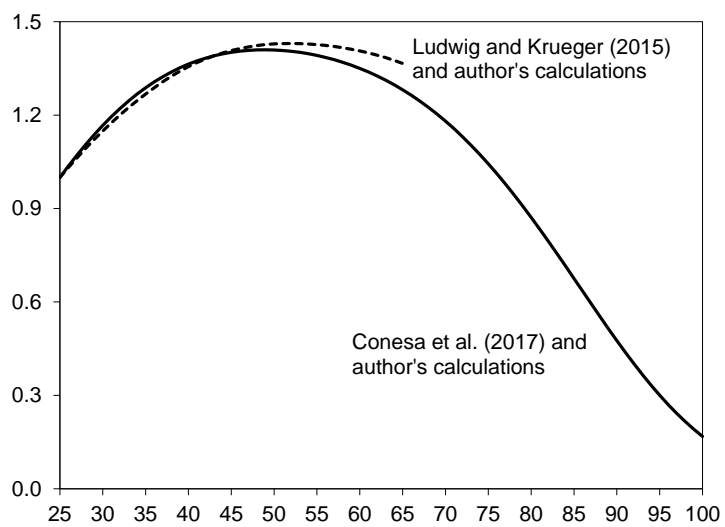


Table 4.1: **Other parameters**

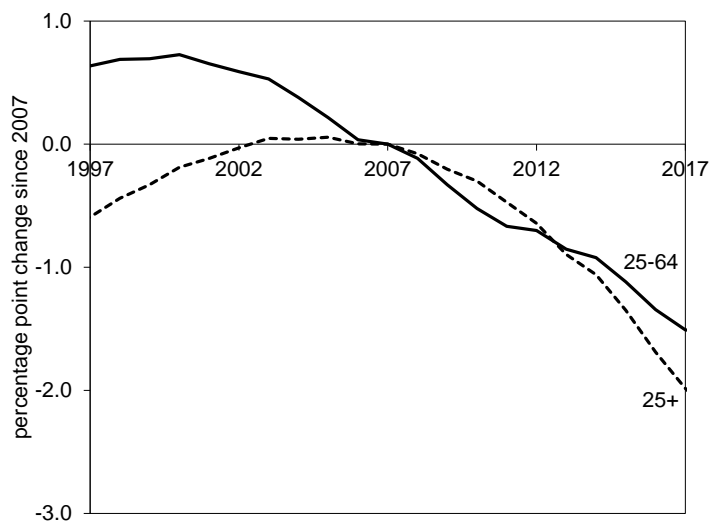
Parameter	Description	Value
$\alpha$	Capital share	0.360
$\delta$	Depreciation rate	0.045
$\sigma$	IES=2	2.742
$\gamma$	Consumption share	0.574
$\beta$	Discount rate	0.940
$r$	Interest rate	0.060
$\rho_\eta$	Persistence	0.953
$\sigma_\eta^2$	Var. persistent	0.060

Table 4.1 shows the remaining parameters. Standard estimates are used for the capital share  $\alpha$ , depreciation rate  $\delta$ , and  $\sigma$  which implies an intertemporal elasticity of substitution equal to 2. The consumption share in the utility function  $\gamma$  is in the range estimated in French (2005). Estimates for the earnings process are taken from Guvenen, Ozkan, and Song (2014).

## 4.5 Results

Figure 4.11 shows the change in employment to population from its 2007 level for the period from 1997 to 2007. From 2007, employment to population for ages 25-64 decreases by more than 1 percentage point; employment to population for ages 25 plus decreases by almost 2 percentage points. The former is consistent with the numbers from the counterfactual exercise. For the latter, the decrease is smaller than what was observed in the counterfactual exercise. This could be driven by increasing survival rates.

Figure 4.12 shows the change in GDP to population (25-64) and labor productivity (GDP to employment, 25-64) for the period from 1997 to 2017. The labor productivity increases, however, there is a decrease in GDP to population (25-64). This divergence is consistent with what was observed for the U.S. economy in the introduction. Labor productivity increases because with an aging population, more productive households are employed. GDP to population (25-64) decreases because of an opposing effect. With an aging population, there are more households who are not employed.

Figure 4.11: **Employment to population (model)**Figure 4.12: **GDP and labor productivity (model)**

## 4.6 Conclusion

Since 2007, employment to population has not recovered; employment to labor force has recovered; and labor productivity recovered faster than GDP. This paper showed that the aging population experienced by the U.S. economy can jointly explain these 3 facts. An implication of this result is that the U.S. economy will not recover to its trend prior to the Great Recession.

# References

- Arellano, C., Bai, Y., and P. J. Kehoe (2012), "Financial Frictions and Fluctuations in Volatility," Federal Reserve Bank of Minneapolis Staff Report 466.
- Athreya, K. (2001), "The growth of unsecured credit: Are we better off?" *Federal Reserve Bank of Richmond Economic Quarterly*, 87(3), 11-33.
- Athreya, K. (2004), "Shame as it ever was: Stigma and personal bankruptcy," *Federal Reserve Bank of Richmond Economic Quarterly*, 90(2), 1-19.
- Athreya, K. (2005), "Equilibrium models of personal bankruptcy: A survey," *Federal Reserve Bank of Richmond Economic Quarterly*, 91(2), 73-98.
- Athreya, K., J. M. Sanchez, X. S. Tam, and E. R. Young (2014), "Labor market upheaval, default regulations, and consumer debt," *Review of Economic Dynamics*, 18(1), 32-52.
- Athreya, K., J. M. Sanchez, X. S. Tam, and E. R. Young (2015), "Bankruptcy and Delinquency in a Model of Unsecured Debt," Working Paper.
- Athreya, K., X. S. Tam, and E. R. Young (2012), "A quantitative theory of information and unsecured credit," *American Economic Journal: Macroeconomics*, 4(3), 153-183.
- Ausubel, L. M. (1991), "The failure of competition in the credit card market," *American Economic Review*, 81(1), 50-81.
- Autor, D. H and D. Dorn (2013), "The Growth of Low-Skill Service employment and the Polarization of the US Labor Market," *American Economic Review*, 103(5), 1553-1597.
- Autor, D. H, F. Levy, and R. J. Murnane (2003), "The Skill Content of Recent Technological Change: An Empirical Exploration," *Quarterly Journal of Economics*, 118(4), 1279-1334.

- Bethune, Z. (2015), "Consumer Credit, Unemployment, and Aggregate Labor Market Dynamics", Working Paper.
- Bethune, Z., G. Rocheteau, and P. Rupert (2015), "Aggregate Unemployment and Household Unsecured Credit", *Review of Economics Dynamics*, 18(1), 77-100.
- Bloom, N., Floetotto, M., Saporta-Eksten I., Jaimovich, N., and S. Terry (2011), "Really Uncertain Business Cycles," NBER Working Paper.
- Board of Governors of the Federal Reserve System (2014), "Report to the Congress on the Profitability of Credit Card Operations of Depository Institutions," Report to Congress
- Chatterjee, S., D. Corbae, K. P. Dempsey, and J. Rios-Rull (2006), "A theory of credit scoring and the competitive pricing of default risk," Working Paper.
- Chatterjee, S., D. Corbae, M. Nakajima, and J. Rios-Rull (2006), "A quantitative theory of unsecured consumer credit with a risk of default," *Econometrica*, 75(6), 1525-1589.
- Clementi, G. L. and B. Palazzo (2014), "Entry, Exit, Firm Dynamics, and Aggregate Fluctuations," NBER Working Paper.
- Conesa, J. C., T. J. Kehoe, and K. K. Ruhl (2007), "Modeling Great Depressions: The Depression in Finland in the 1990s," *Quarterly Review*, Federal Reserve Bank of Minneapolis, issue Nov, 16-44.
- Decker, R., Haltiwanger, J., Jarmin, R., and J. Miranda (2014), "The Role of Entrepreneurship in the US Job Creation and Economic Dynamics," *Journal of Economic Perspectives*, 28 (3), 3-24.
- Droz, L. A., and J. B. Nosal (2008), "Competing for customers: A search model of the market for unsecured credit," Working Paper.
- Eaton, J. and M. Gersovitz (1981), "Debt with potential repudiation: Theoretical and empirical analysis," *Review of Economic Studies*, 48(2), 289-309.
- Eden, M. and P. Gaggl (2015), "On the welfare implications of automation," Working Paper.
- Elsby, M. W. L., Hobijn, B., Sahin, A., and R. G. Valletta (2011), "The Labor Market in the Great Recession: An Update," Brookings Panel on Economic Activity.

- Fay, S., E. Hurst, and M. J. White (2002), "The household bankruptcy decision," *American Economic Review*, 92(3), 706-718.
- Foote, C. L and R. W. Ryan (2014), "Labor-Market Polarization Over the Business Cycle," NBER Working Paper.
- Greenwood, J, Z. Hercowitz, and P. Krusell (1997), "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, 87(3), 342-362.
- Gross, D. B. and N. S. Souleles (2002), "An empirical analysis of personal bankruptcy and delinquency," *Review of Financial Studies*, 15(1), 319-347.
- Guvenen, F., S. Ozkan, and J. Song (2014), "The nature of countercyclical income risk," *Journal of Political Economy*, 122 (5), 621-660.
- Haltiwanger, J., Jarmin, R., and J. Miranda (2013), "Anemic Job Creation and Growth in the Aftermath of the Great Recession: Are Home Prices to Blame?" Business Dynamics Statistics Briefing.
- Haltiwanger, J., Jarmin, R., and J. Miranda (2012), "Where Have All the Young Firms Gone?" Business Dynamics Statistics Briefing.
- Herkenhoff, K. (2015), "The impact of consumer credit access on unemployment," Working Paper.
- Hodrick, R. and E. C. Prescott (1997), "Postwar U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit, and Banking*, 29(1), 1-16.
- Hopenhayn, H. and Rogerson, R. (1993), "Job turnover and policy evaluation: a general equilibrium analysis," *Journal of Political Economy*, 101, 915-938.
- Jaimovich, N. and H. E. Siu (2014), "The Trend is the Cycle: Job Polarization and Jobless Recoveries," NBER Working Paper.
- Jarmin, R. S. and J. Miranda (2002), "The Longitudinal Business Database," U.S. Census Bureau.
- Karabarbounis, L. and B. Neiman (2014), "The global decline of the labor share," *Quarterly Journal of Economics*, 129(1), 61-103.



- Khan, A. and Thomas, J.K. (2003), "Nonconvex factor adjustment in equilibrium business cycle models: Do nonlinearities matter?" *Journal on Monetary Economics*, 50, 331-360.
- Krusell, P., L. E. Ohanian, J. Rios-Rull, and G. L. Violante (2000), "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica*, 68(5), 1029-1053.
- Krusell, P. and A. A. Smith (1998), "Income and Wealth Heterogeneity in the Macroeconomy", *Journal of Political Economy*, 106 (5), 867-896.
- Kydland, F. E. and Prescott E. C. (1982), "Time to Build and Aggregate Fluctuations," *Econometrica*, 50 (6), 1345-1370.
- Lee, Y. and T. Mukoyama (2013), "Entry, Exit, and Plant-level Dynamics over the Business Cycle," Working Paper.
- Lehn, C. (2014), "Labor market polarization, the decline of routine work, and technological change: A quantitative analysis," Working Paper.
- Livshits, I., J. MacGee, and M. Tertilt (2007), "Consumer Bankruptcy: A Fresh Start," *The American Economic Review*, 97(1), 402-418.
- Livshits, I., J. MacGee, and M. Tertilt (2010), "Accounting for the rise in consumer bankruptcies," *American Economic Journal: Macroeconomics*, 2(2), 165-193.
- Livshits, I., J. MacGee, and M. Tertilt (2015), "The democratization of credit and the rise in consumer bankruptcies," *Review of Economics Studies*, (forthcoming).
- Luttmer, E. (2010), "Models of Growth and Firm Heterogeneity," *Annual Reviews of Economics*, 2, 547-576.
- McKinley, V. (1997), "Ballooning bankruptcies: Issuing blame for the explosive growth," *Regulation*, 20(4), 3340.
- Mehrotra, N. and D. Sergeyev (2013), "Financial Shocks and Job Flows," Working Paper.
- Meyer, P. B and A. M. Osborne (2005), "Proposed category system for 1960-2000 Census occupations," BLS Working Paper.

- Morin, M. (2014), "Computer Adoption and the Changing Labor Market," Working Paper.
- Narajabad, B. N. (2012), "Information technology and the rise of household bankruptcy," *Review of Economic Studies*, 15(4), 526-550.
- Rogerson, R. and J. Wallenius (2009), "Micro and macro elasticities in a life cycle model with taxes," *Journal of Economic Theory*, 144, 2277-2292.
- Rogerson, R. (2014), "Comments on Foote and Ryan by Richard Rogerson."
- Rouwenhorst, K. G. (1995), "Asset Pricing Implications of Equilibrium Business Cycle Models," *Princeton University Press*, 294-330.
- Ruttenberg, A. J. (2012), "The totality of what circumstances?" Looney and Grossman LLP.
- Sanchez, J. M. (2012), "The IT revolution and the unsecured credit market," Working Paper.
- Shimer, R. (2008), "The Probability of Finding a Job," *The American Economic Review*, 98 (2), 268-273.
- Siemer, M. (2014), "Firm Entry and Employment Dynamics in the Great Recession," Working Paper.
- Storesletten, K., C. Telmer, and A. Yaron (2004), "Consumption and risk sharing over the life cycle," *Journal of Monetary Economics*, 51(3), 609-633.
- Waddle, A. (2013), "Globalization and the Changing Shape of Labor Recoveries," Working paper.

## Appendix A

# The Rise in Unsecured Credit and Consumer Bankruptcies

### A.1 Data

Table A.1: Variable and sources

unsecured credit <sup>1</sup>	Board of Governors of the Federal Reserve System (FRB)
gross domestic product	Bureau of Economic Analysis
consumer bankruptcies <sup>2</sup>	McKinley (1997) and American Bankruptcy Institute
working-age population	World Bank World Development Indicators
unsecured credit access <sup>3</sup>	Survey of Consumer Finances (SCF)
charge-off rate	Ausubel (1991) and FRB
average credit card interest rate	FRB
3-month treasury bill secondary market rate	FRB
dispersion of spreads	SCF and Consumer Financial Protection Bureau

### A.2 Borrowing constraint and financial intermediary budget constraint

The borrowing constraint  $\bar{b}_t(\lambda)$  satisfies the following condition:

$$\frac{E_\lambda[1 - d(\bar{b}(\lambda), \lambda')]}{1 + r_{t+1} + \tau} = \mathfrak{q}$$

Aggregate capital  $K_{t+1}$  is pinned down using the following budget constraint of the financial intermediary.

$$\kappa \int v_t(db \times d\lambda) \Omega_t(db \times d\lambda \times N) - \int b(1 - d(b, \lambda)) \Omega_t(db \times d\lambda \times c) + K_{t+1} = K_t(1 + r) - \int q_t(b', \lambda) b'(b, \lambda, c) \Omega_t(db \times d\lambda \times c)$$

### A.3 Computation algorithm for stationary equilibrium

1. Guess ratio for  $K/L$ , which gives prices for  $r$  and  $w$  from the production function for  $Y$ 
  - (a) Guess value functions for households  $V_0(b, \lambda, i)$ , profit functions for lenders  $\Pi_0(b, \lambda)$ , policy functions for borrowing  $b'_0(b, \lambda, i)$ , and default decision  $d_0(b, \lambda)$
  - (b) Back out bond price schedule  $q(b', \lambda)$
  - (c) Solve for new profit functions for lenders  $\Pi_1(b, \lambda)$
  - (d) Back out  $p(b', \lambda')$  from the zero profit condition
  - (e) Solve for new value functions  $V_1(b, \lambda, i)$  and policy functions,  $b'_1(b, \lambda, i)$  and  $d_1(b, \lambda)$
  - (f) Update guesses in (a); repeat (a) to (e) until convergence
2. Given policy functions, simulate and solve for stationary distribution of households  $\Omega$
3. Back out aggregates from stationary distribution and update  $K/L$ ; repeat 1 to 3 until convergence

### A.4 Computation algorithm for transition

1. Solve for initial and final stationary equilibria
2. Guess a sequence for  $K/L$  and back out a sequence of prices for  $r$  and  $w$

3. Use backward induction to solve for bond price schedule, value functions, policy functions, profit functions, and credit access probabilities
4. Given policy functions and initial distribution  $\Omega$ , simulate the economy
5. Back out new guesses for sequence for  $K/L$ ; repeat 1 to 5 until convergence

## A.5 Additional Tables

Table A.2: **Alternative explanations: parameters calibrated**  
**Decrease in the cost of bankruptcy, stigma  $\chi$**

$\chi_{1970}$	Stigma 1970	<i>Bankruptcies per WAP</i> $_{1970} = 0.09$ percent	2.7127
$\chi_{2004}$	Stigma 2004	<i>Maximum bankruptcies per WAP</i>	0
$\beta$	Discount rate	<i>Annual capital to output</i> $_{1970} = 3$	0.9186
$\theta$	Labor productivity	<i>GDP per working age person</i> $_{1970} = 1$	0.5386
$\delta$	Depreciation rate	<i>Interest rate</i> $_{1970} = 4.00$ percent	0.0813
$\alpha_c$	Borrower share	<i>Unsecured credit to GDP</i> $_{1970} = 0.40$ percent	0.3911
$A$	Matching efficiency	<i>Unsecured Credit access</i> $_{2004} = 71.47$ percent	0.0045

**Decrease in the lending fee  $\tau$**

$\beta$	Discount rate	<i>Annual capital to output</i> $_{1970} = 3$	0.9190
$\theta$	Labor productivity	<i>GDP per working age person</i> $_{1970} = 1$	0.5390
$\delta$	Depreciation rate	<i>Interest rate</i> $_{1970} = 4.00$ percent	0.0804
$A$	Matching efficiency	<i>Unsecured Credit access</i> $_{2004} = 71.47$ percent	0.0044

**Decrease in stigma  $\chi$  and lending fee  $\tau$**

$\chi_{1970}$	Stigma 1970	<i>Bankruptcies per WAP</i> $_{1970} = 0.09$ percent	2.2185
$\chi_{2004}$	Stigma 2004	<i>Maximum bankruptcies per WAP</i>	0
$\beta$	Discount rate	<i>Annual capital to output</i> $_{1970} = 3$	0.9186
$\theta$	Labor productivity	<i>GDP per working age person</i> $_{1970} = 1$	0.5389
$\delta$	Depreciation rate	<i>Interest rate</i> $_{1970} = 4.00$ percent	0.0850
$\alpha_c$	Borrower share	<i>Unsecured credit to GDP</i> $_{1970} = 0.40$ percent	0.3025
$A$	Matching efficiency	<i>Unsecured Credit access</i> $_{2004} = 71.47$ percent	0.0054

**Increase in the lender's information (decrease in  $\phi$ )**

$\phi_{1970}$	Weight on false type 1970	<i>Bankruptcies per WAP</i> $_{1970} = 0.09$ percent	0.8072
$\phi_{1997}$	Weight on false type 2004	<i>Bankruptcies per WAP</i> $_{2004} = 0.56$ percent	0.0074
$\beta$	Discount rate	<i>Annual capital to output</i> $_{1970} = 3$	0.9189
$\theta$	Labor productivity	<i>GDP per working age person</i> $_{1970} = 1$	0.5390
$\delta$	Depreciation rate	<i>Interest rate</i> $_{1970} = 4.00$ percent	0.0814
$\alpha_c$	Borrower share	<i>Unsecured Credit to GDP</i> $_{1970} = 0.40$ percent	0.4314
$A$	Matching efficiency	<i>Unsecured credit access</i> $_{2004} = 71.47$ percent	0.0115

## Appendix B

# Trend and Cycle of Low-Skilled Manufacturing Employment

### B.1 Construction of low-skilled manufacturing employment

Annual data on employment by skill and manufacturing from 1968 to 2014 is sourced from BLS CPS. High-skilled manufacturing is stable at 2-3 percent ( Figure 2.3 ). Quarterly data on manufacturing employment from 1948 to 2015 is sourced from BLS CES. CPS annual data is used to adjust CES quarterly data to construct high-skilled manufacturing employment. Given CPS data for a particular year, quarterly high-skilled manufacturing employment is constructed as

$$\frac{high - skilled\ manufacturing_{cps}}{total\ employment_{cps}} \times total\ employment_{ces}.$$

The adjustment ratio for years prior to 1968 is kept at the 1968 level. The adjustment ratio for 2015 is kept at the 2014 level. Quarterly low-skilled manufacturing employment is calculated given the constructed high-skilled manufacturing employment, data on non-manufacturing employment, and total employment.

## B.2 Construction of series for capital structure and capital equipment

Series for capital structure and capital equipment are constructed using the perpetual inventory method as in Conesa, Kehoe, and Ruhl (2007). Initial capital structure  $k_{s1948}$  is chosen such that

$$\frac{k_{s1948}}{y_{1948}} = \frac{1}{40} \sum_{t=1948}^{1957} \frac{k_{st}}{y_t}.$$

Initial capital equipment  $k_{e1948}$  is chosen such that

$$\frac{q_{1948}k_{e1948}}{y_{1948}} = \frac{1}{40} \sum_{t=1948}^{1957} \frac{q_t k_{et}}{y_t}.$$

Note the initial level of capital structure and capital equipment have been chosen outside the period of analysis so that they don't have a direct impact on results.

## B.3 Equivalence between investment-specific growth and relative price of equipment

Exogenous relative equipment price is equivalent to exogenous investment-specific productivity growth. Consider another economy where

$$y = k_s^\alpha [\mu(z l_m)^\eta + (1 - \mu)(\lambda(z_k \tilde{k}_e)^\rho + (1 - \lambda)(z l_n)^\rho)^{\frac{\eta}{\rho}}]^{\frac{1-\alpha}{\eta}}$$

$$x_e = \tilde{k}'_e - (1 - \tilde{\delta}_e) \tilde{k}_e$$

$z_k$  is investment-specific productivity growth and relative price of equipment is 1. Let

$$z_k = 1/q_{-1}$$

$$\tilde{k}'_e = k'_e q$$

$$1 - \tilde{\delta}_e = (1 - \delta_e)q/q_{-1}$$



This economy is equivalent to benchmark economy (results follow directly from first order conditions).

## Appendix C

# Lack of Firm entry and the Slow Recovery of the U.S. Economy after the Great Recession

### C.1 Numerical Solution

Since it is not possible to keep track of the firm distribution  $\Omega$ , we follow Clementi and Palazzo (2014) and assume the following forecasting rules for  $w'$  where

$$\log w' = \alpha_w + \beta_w \log w + \beta_{Z'} \log Z + \beta_{Z'} \log Z'.$$

### C.2 Firm's Approximated Problem

Given an initial guess for the  $\{\alpha_w, \beta_w, \beta_Z, \beta_{Z'}\}$ , the firm uses the law of motion for  $w'$  and solves the following problem,

$$\tilde{V}^f(s, Z, w) = \pi(s, Z, w) + \beta(1 - \eta)E_Z \tilde{V}^f(s, Z', w')$$

where  $\pi(s, Z, w)$  is given by

$$\max_{l_f(s, Z, w)} sZl_f(s, Z, w)^\theta - w(Z)l_f(s, Z, w)$$

### C.3 Algorithm

1. given a guess for  $\{\alpha_w, \beta_w, \beta_Z, \beta_{Z'}\}$ , approximate the firms' value function
2. simulate the economy with TFP shocks
3. in the simulation, for every period, we have to solve for  $\{w, \mu\}$  such that the labor market clears and zero profit condition holds in equilibrium. Nelder-Mead along with Newton's method is used to clear the market and ensure that the zero profit condition holds in equilibrium. Note that when the values for  $\{w, \mu\}$  are updated, we have to re-optimize decision rules for both the firm and household. However, we still use the laws of motion and value function from step 2 to forecast in the firm's problem.
4. Given the results of the simulation, we can use OLS to get new estimates for  $\{\alpha_w, \beta_w, \beta_Z, \beta_{Z'}\}$ . Then we go back to step 2 until parameters converge.

The initial guess for  $\alpha_w$  is  $\log w$  where  $w$  is the wage rate in the economy without aggregate uncertainty. The initial guesses for  $\{\beta_w, \beta_Z, \beta_{Z'}\}$  are set to 0. We get an r-square of .999999 for the law of motion. We first start by simulating the economy for 750 periods where we ignore the first 250 periods for OLS estimates of  $\{\alpha_w, \beta_w, \beta_Z, \beta_{Z'}\}$ . Once the parameters have converged, we do a check where we simulate the economy for 3,500 periods where we ignore the first 500 periods.